The power of the CAS!

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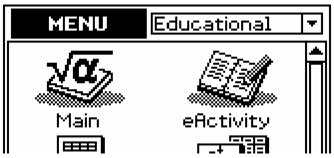
Question: For the equation $y = (x - 1)^2(x - a)$, where $a > 1$, find the exact value of a such that the local minimum point lies on the line $y = -4x$
What are the exact coordinates of this minimum point?

Answer: What can we to do? 'A plan of attack!'

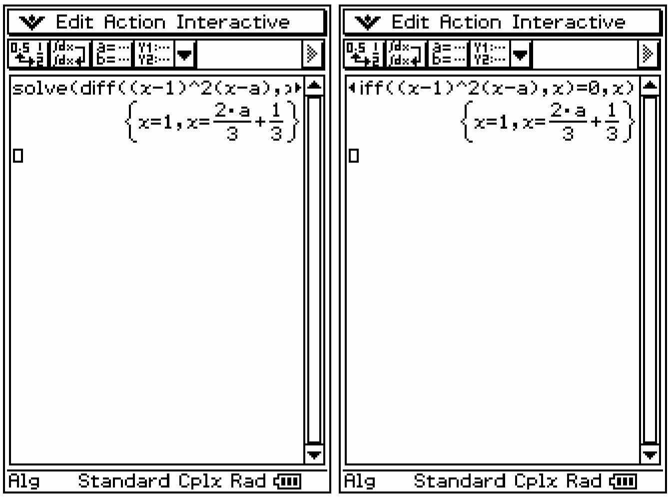
1. Differentiate $y = (x - 1)^2(x - a)$ and set $^{dy}/_{dx} = 0$, as the local minimum point will satisfy this condition.
2. Find the intersection points of $y = (x - 1)^2(x - a)$ and $y = -4x$, taking the solution that is greater than 1.
3. Using $y = (x - 1)^2(x - a)$, solve for a using the co-ordinate point (a,0), taking the solution that x >1.
4. Check that this is the solution.

ClassPad 300 screen dumps to illustrate.

Enter the '**Main**' icon.

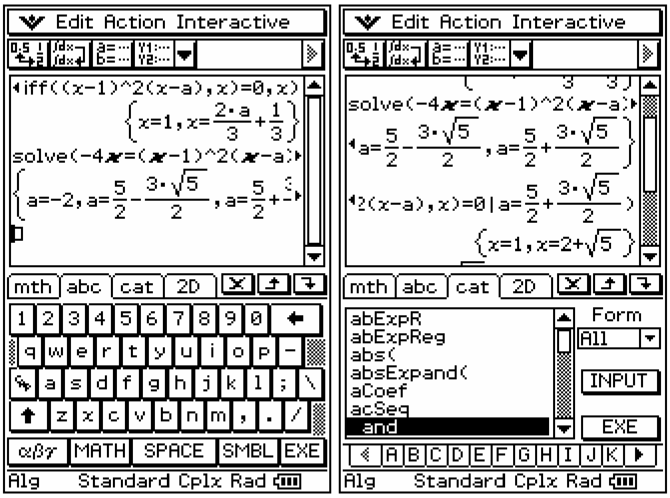


1. Differentiate $y = (x - 1)^2(x - a)$ and set dy/dx = 0, to find the x-value of the turning points is a local minimum point.



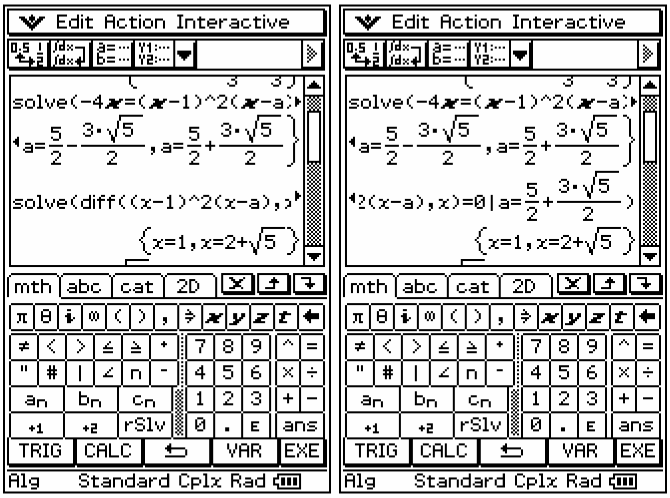
Due to the nature of the cubic defined $y = (x - 1)^2(x - a)$, $a > 1$, there is a double root at x = 1, a maximum, hence the minimum will occur at a point on the interval $1 < x < a$. The required solution is $x = {}^{2a}/_3 + {}^1/_3$ because of the constraint $a > 1$ and is a local minimum point.

2. Find the intersection points of the equations: $y = (x - 1)^2(x - a)$ and $y = -4x$, taking the solution that has x >1.



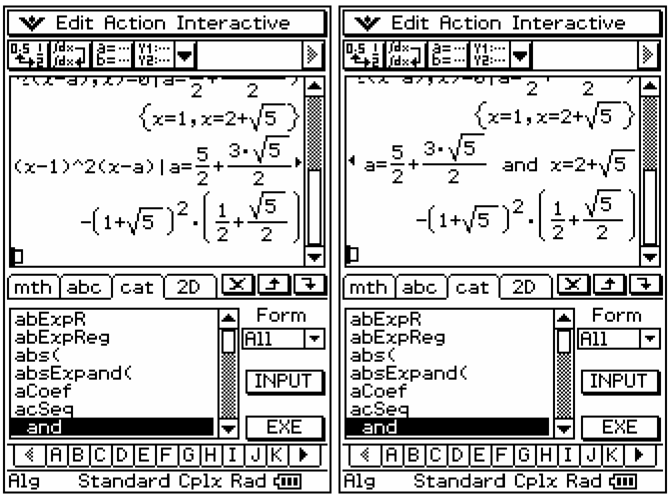
Required solution is $a = {}^5/_2 + {}^{3\sqrt{5}}/_2$, because of the constraint $a > 1$.

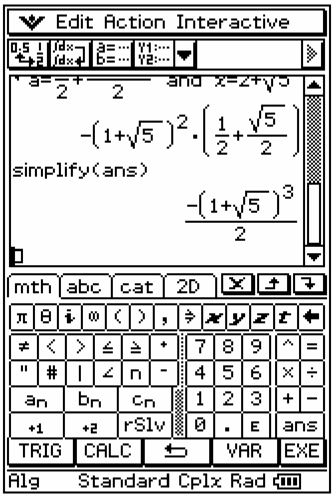
3. Using $y = (x - 1)^2(x - a)$, solve for a using the co-ordinate point (a,0), taking the solution that x >1.

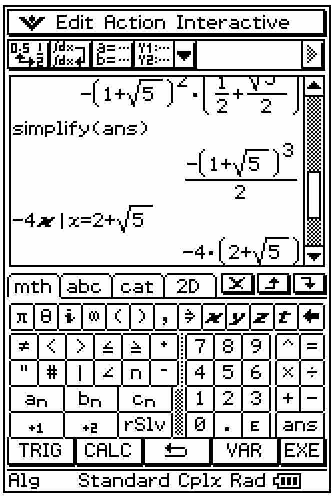


Required solution is $x = 2 + \sqrt{5}$, because of the constraint $x > 1$, from original interpretation of the cubic graph.

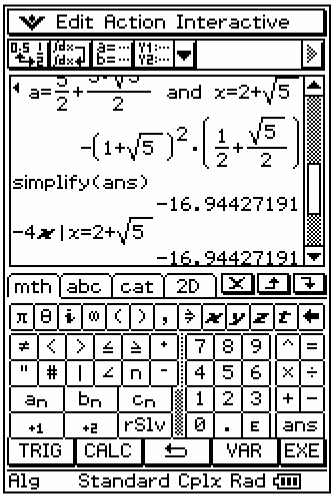
4. Check that this is the solution.







Or as decimals and comparing the two calculations



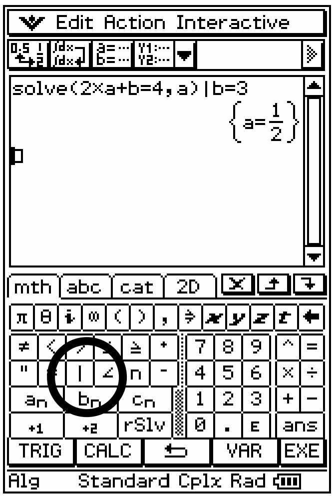
Answer: When $a = {}^{5}/_{2} + {}^{3\sqrt{5}}/_{2}$ the coordinates of the minimum point is $(\ 2 + \sqrt{5}\ ,\ -\tfrac{1}{2}\ (1 + \sqrt{5})^3\ )$ ☺

Something else to ponder on!

Using the '|' symbol via Math keypad and OPTN.

A quick example to whet your appetite!

Calculate 2a + b = 4 given b = 3



Example 2:

Given $f(x) = ax^2+bx+c$

Solve for a and b in terms of c, given that f(1) = 3 and f '(4) = 1

By pencil and paper method:

a + b + c = 3 by substitution of x = 1

8a + b = 1 by differentiating and then substitution of x = 4

NOW

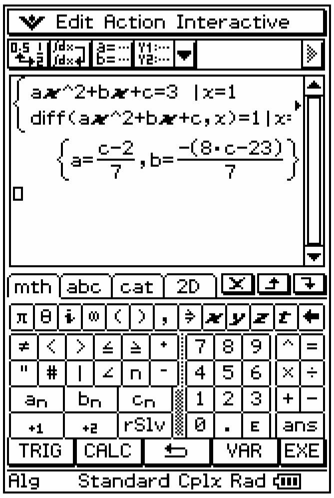
Solving for 'a' and 'b'

7a – c = -2 and 8a + 8b + 8c = 24

8a + b = 1

7b + 8c = 23 giving

$a = {}^{(-2 + c)}/_{7}$ and $b = {}^{(23 - 8c)}/_{7}$



Student working this example using CAS:

I need to Solve ax^2+bx+c=3|x=1 and diff(ax^2+bx+c,x)=1|x=4

ANSWER: a = (c-2)/7 and b = -(8c-23)/7

OR

$\begin{cases} \text{ax^2+bx+c=3|x=1} \\ \text{diff(ax^2+bx+c,x)=1|x=4 a,b} \end{cases}$

a = (c-2)/7 and b = -(8c-23)/7

What would you expect to see in a student piece of work? **'A plan of attack!'** ☺