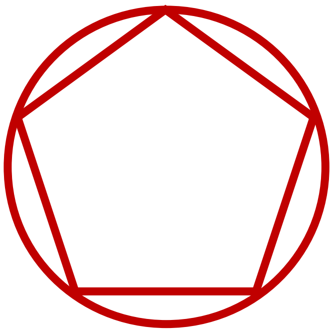
Star Polygons on the Classpad

This resource was written by Derek Smith with the support of CASIO New Zealand. It may be freely distributed but remains the intellectual property of the author and CASIO.

Star polygons are constructed by connecting evenly-spaced dots on a circle. The segments formed must all be the same length. If the segments do not cross each other, the figure formed is a regular polygon. If the segments cross each other, the polygon formed is called a **star polygon**.

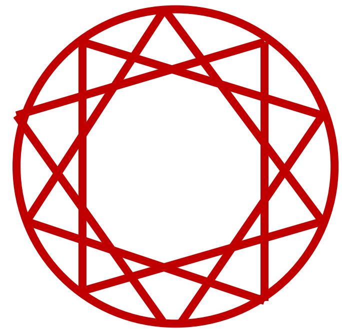


$\left\{ \begin{matrix} 5 \\ 1 \end{matrix} \right\}$

Pentagon
Pentagon inside

$\left\{ \begin{matrix} 5 \\ 2 \end{matrix} \right\}$

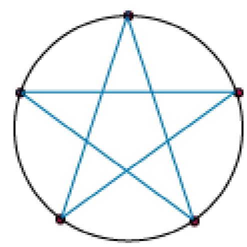
Star polygon
Pentagon inside



$\left\{ \begin{matrix} 10 \\ 3 \end{matrix} \right\}$

Star polygon
Decagon inside

A 5-pointed star (pentagram) can be formed by connecting every second dot on a circle with 5 evenly-spaced dots, as is shown at the right. The Schläfi notation for this star polygon is given below the diagram. If every dot is connected in order, the resulting figure is a regular pentagon. Note that, if every third dot were connected, the resulting star polygon would be the same the **star polygon** created if every second dot were connected.

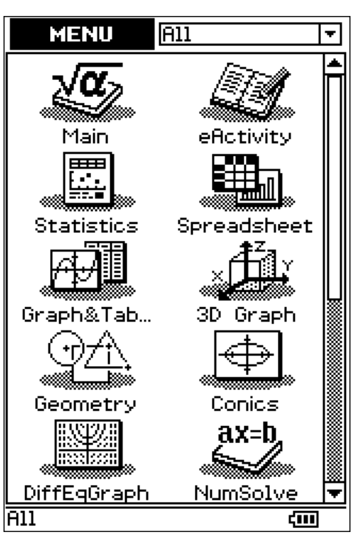


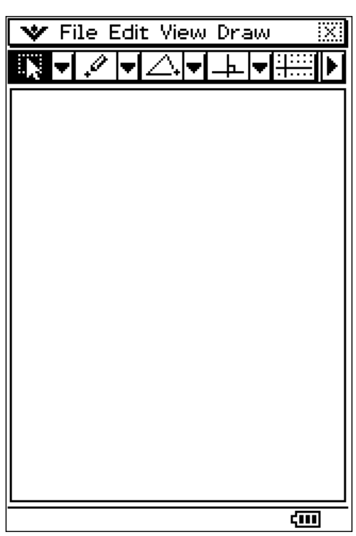
There are only two figures that can be shown on a 5-dot circle. The 5-point star and a regular pentagon.

$\left\{ \begin{matrix} 5 \\ 1 \end{matrix} \right\}$

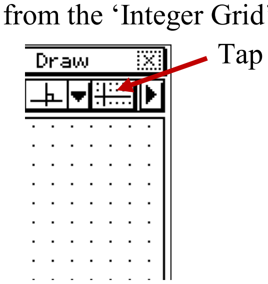
Notice that: $\left\{ \begin{matrix} 5 \\ 1 \end{matrix} \right\} = \left\{ \begin{matrix} 5 \\ 4 \end{matrix} \right\}$ and a $\left\{ \begin{matrix} 5 \\ 2 \end{matrix} \right\} = \left\{ \begin{matrix} 5 \\ 3 \end{matrix} \right\}$

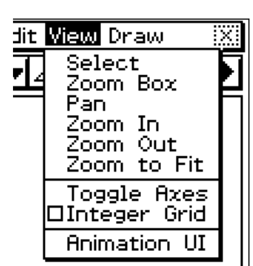
Creating these on your **ClassPad** star polygons. From the MENU enter into the **Geometry** icon.



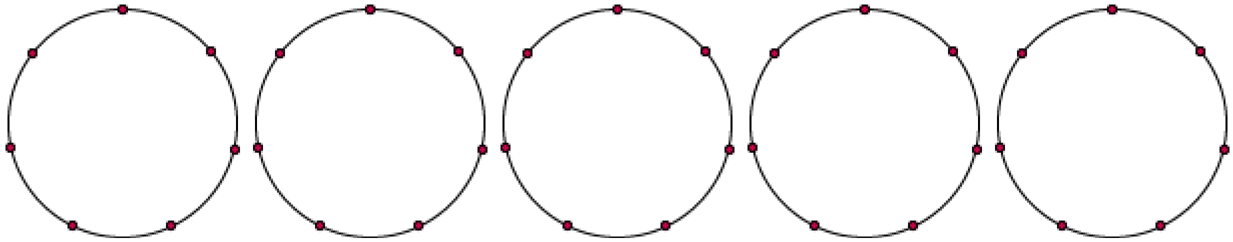


Remove the integer grid and the x and y axes by tapping on the icon on the top left and in the **View** drop down box make sure that the tick is removed from the 'Integer Grid' box.

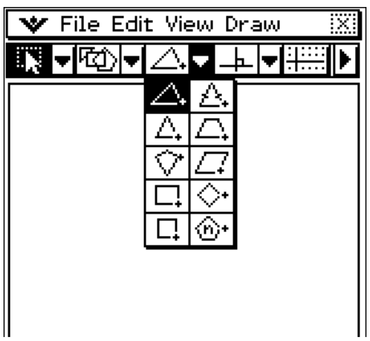


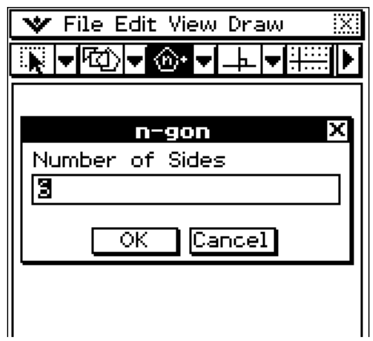


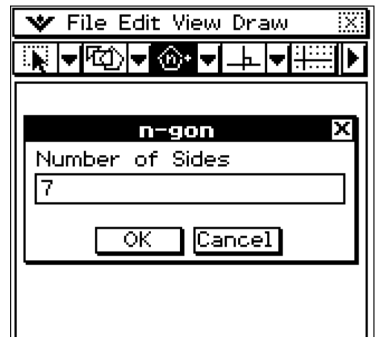
Using the 7-dot circles below or create the 7-sided polygon (heptagon) on your ClassPad to construct the five star polygons and write the code below each. **Why are there only 3 and not 5 different star polygons?**



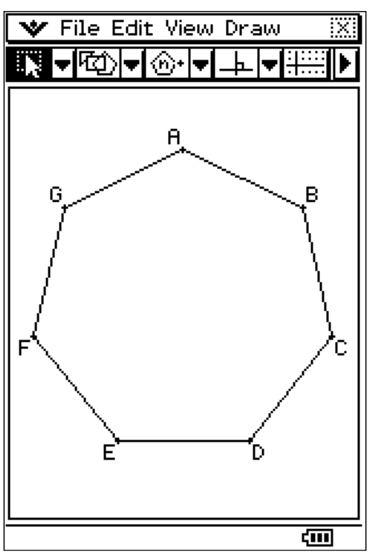
To construct a polygon select the drop down menu indicated in the screen capture below.

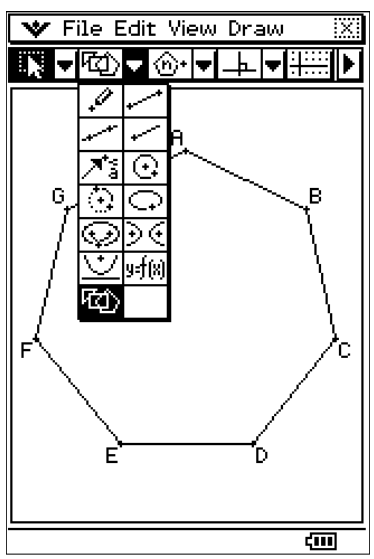


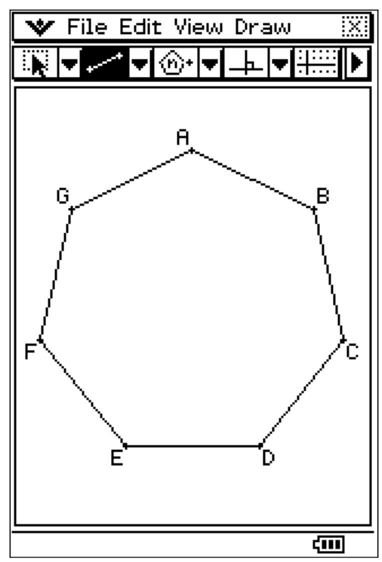




The default is 6-sides, change this value in the 'Number of Sides' box to 7.





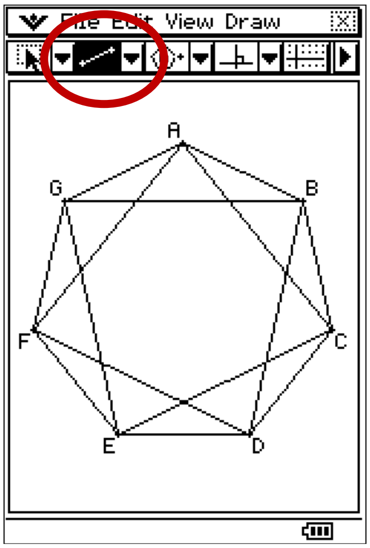


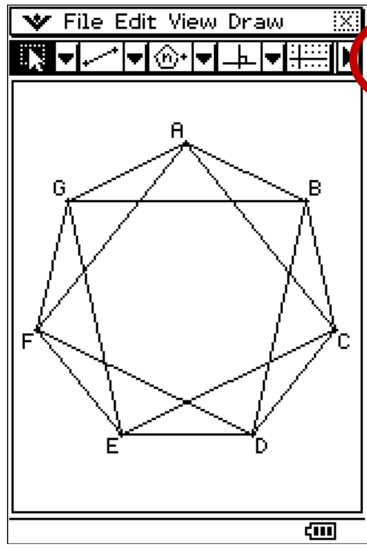
You will be returned to the working geometry page. Now, tap your stylus on the screen and the 7-gon will appear with its vertices labeled A, B, C, D, E, F and G. This is the $\left\{ \begin{matrix} 7 \\ 1 \end{matrix} \right\}$ which is a heptagon and is **NOT** a **star polygon.**

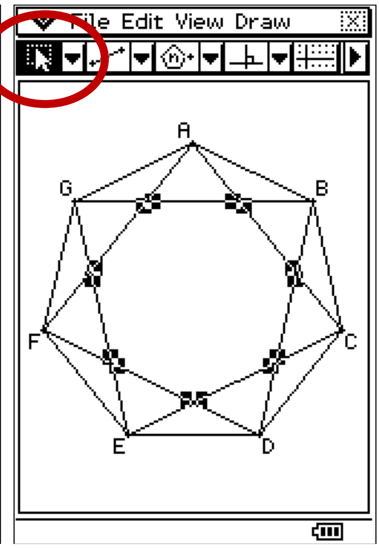
Create a $\left\{ \begin{matrix} 7 \\ 2 \end{matrix} \right\}$ by constructing line segments starting from vertex A and going every 2^{nd} vertex. This is illustrated on the following page.

Line segment tool

To delete the line segments make sure you select the 'pointer'. Select the line segments.

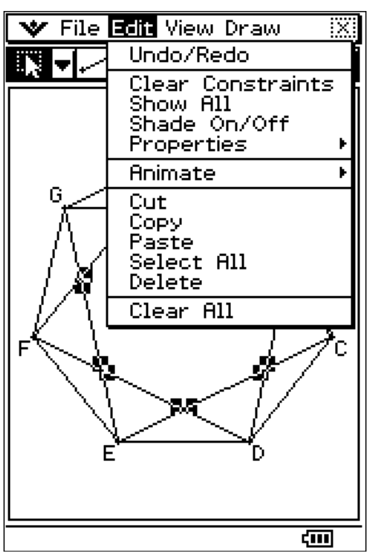


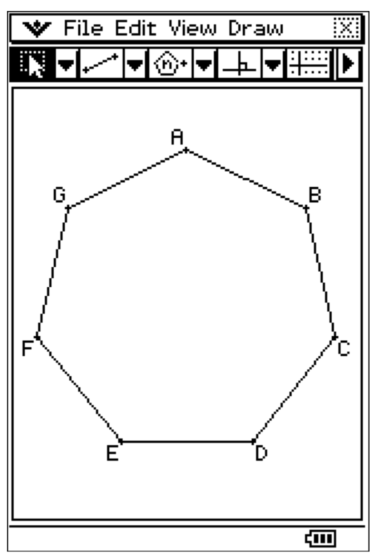


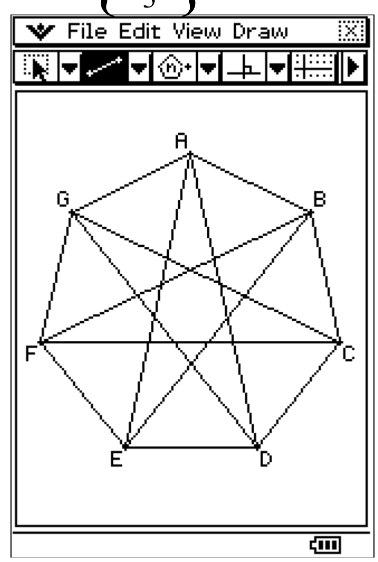


Now, tap on the Edit drop down. And select '**detete**'.

Start again from the heptagon to begin creating the $\left\{ \begin{matrix} 7 \\ 3 \end{matrix} \right\}$ star polygon.



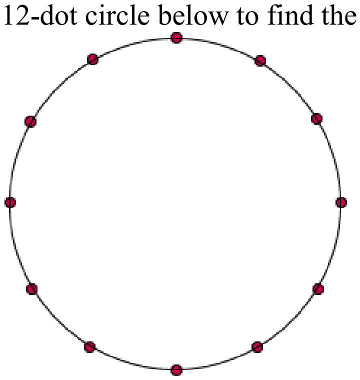


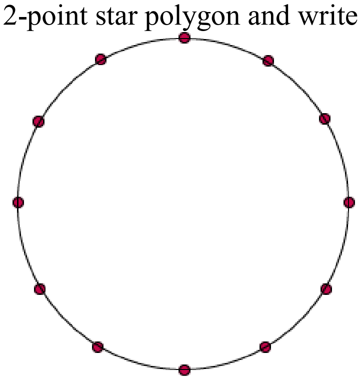


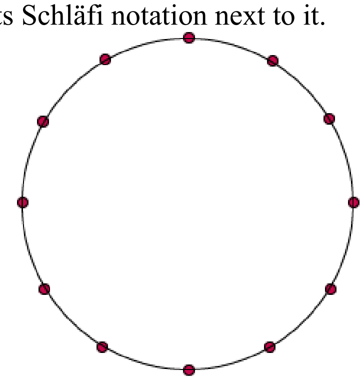
What would a $\left\{ \begin{matrix} 7 \\ 4 \end{matrix} \right\}$, $\left\{ \begin{matrix} 7 \\ 5 \end{matrix} \right\}$ and $\left\{ \begin{matrix} 7 \\ 6 \end{matrix} \right\}$ look like?

What pairs of star polygons can you see create the same?

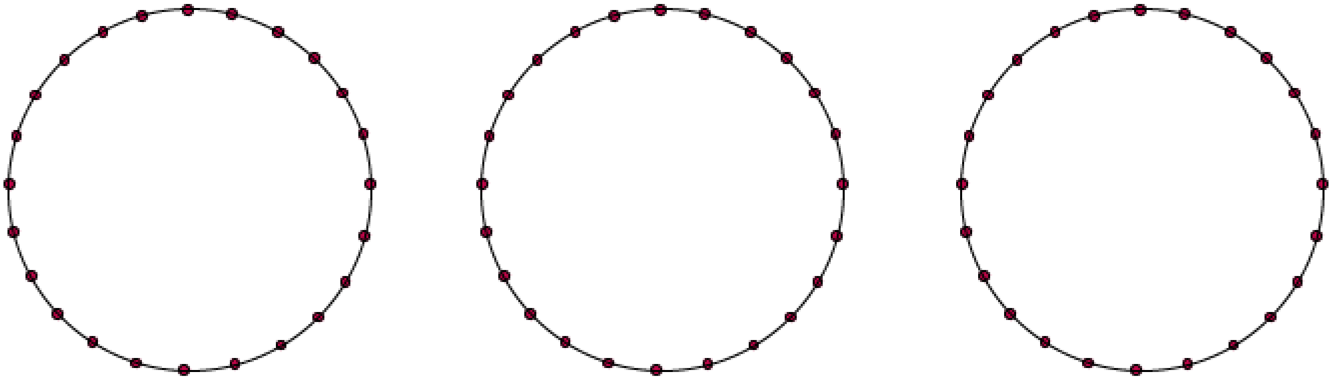
Use the 12-dot circle below to find the 12-point star polygon and write its Schläfi notation next to it.







Use the 24-dot circles below or your ClassPad to construct all of the star polygons. Write the Schläfi notation for each star polygon next to it. Place the name of the polygon in the table following the last circle. Describe each as a star polygon or a regular polygon, and give the number of sides or points.



Describe each as a star polygon, giving the number of points, or as a regular polygon, giving the number of sides is 18.

$\left\{ \begin{matrix} 18 \\ 1 \end{matrix} \right\}$ $\left\{ \begin{matrix} 18 \\ 2 \end{matrix} \right\}$ $\left\{ \begin{matrix} 18 \\ 3 \end{matrix} \right\}$ $\left\{ \begin{matrix} 18 \\ 4 \end{matrix} \right\}$ $\left\{ \begin{matrix} 18 \\ 5 \end{matrix} \right\}$

$\left\{ \begin{matrix} 18 \\ 6 \end{matrix} \right\}$ $\left\{ \begin{matrix} 18 \\ 7 \end{matrix} \right\}$ $\left\{ \begin{matrix} 18 \\ 8 \end{matrix} \right\}$ $\left\{ \begin{matrix} 18 \\ 9 \end{matrix} \right\}$

Regular and Star Polygons

How many regular polygons can be drawn on a circle with *n* evenly-spaced dots? Fill in the information in the table below.

Dots	3	4	5	6	7	8	9	10	11	12	13
Regular Polygons	______	______	______	______	______	______	______	______	______	______	______
Star Polygons	______	______	______	______	______	______	______	______	______	______	______

Dots	14	15	16	17	18	19	20	21	22	23	24
Regular Polygons	______	______	______	______	______	______	______	______	______	______	______
Star Polygons	______	______	______	______	______	______	______	______	______	______	______

Can you generate a rule from this information you have collected?

What would star polygons look like when you used only the Fibonacci numbers?

$\langle 1, 1, 2, 3, 5, 8, 13, 21, 34, \ldots \rangle = \langle a_{n+2} = a_n + a_{n+1}$ where $a_0 = 1$ and $a_1 = 1 \rangle$ Which pairs are the same?

Examples:

$\left\{ \begin{matrix} 2 \\ 1 \end{matrix} \right\}$ $\left\{ \begin{matrix} 3 \\ 1 \end{matrix} \right\}$ $\left\{ \begin{matrix} 3 \\ 2 \end{matrix} \right\}$ $\left\{ \begin{matrix} 5 \\ 2 \end{matrix} \right\}$ $\left\{ \begin{matrix} 5 \\ 3 \end{matrix} \right\}$

$\left\{ \begin{matrix} 5 \\ 4 \end{matrix} \right\}$ $\left\{ \begin{matrix} 8 \\ 1 \end{matrix} \right\}$ $\left\{ \begin{matrix} 8 \\ 2 \end{matrix} \right\}$ $\left\{ \begin{matrix} 8 \\ 3 \end{matrix} \right\}$ $\left\{ \begin{matrix} 8 \\ 5 \end{matrix} \right\}$ etc.