Recurring Decimals

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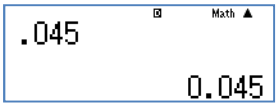
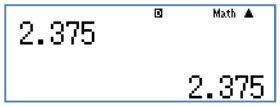
A decimal is a number which includes a decimal point. Every number can be written in a **decimal format**. We shall look at three different types of decimal numbers:

- Exact Decimals
- Recurring Decimals
- Non-Recurring Decimals.

Exact Decimals

An exact decimal is a number that you can write down all the digits. These are the simplest forms of decimals and **0.5**, **0.1, 0.045** and **2.375** are examples.





Recurring Decimals

For something to recur it means to will happen again and again repeatedly. Therefore a recurring decimal is one that goes on forever with some of the digits repeat themselves over and over again.

Often one of the first kinds of recurring decimals we meet in mathematics is $^1/_3$. This can be written as **0.3333333…** where the 3's go on forever.

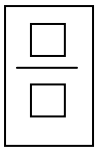
Another example is **0.3444444444444…**. With this example it is interesting to note two things. The first digit after the decimal point, **3** does not repeat. It is possible for more than one digit to recur, in this example it's the digits **5** and **6**, but it could be any number of digits.

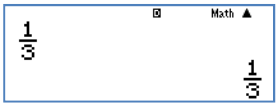
Another example is **0.345656565656…**. With this example it is interesting to note two things. The first two digits after the decimal point, (**3** and **4**) do not repeat. Again it is possible for more than one digit to recur, in this example it's the digits **5** and **6**, but it could be any number of digits.

In order to write recurring decimals in a neater way, mathematicians write the decimal with dots over the recurring digits, or a line or with the dots in this example over the first and last repeating digit for example:

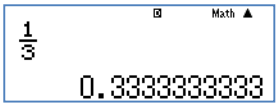
$^1/_3$= **0.333333…** = $0.\dot{3}$ and **0.22103103103…** = $0.22\dot{1}0\dot{3}$ or **0.345656565656… = 0.3$\overline{456}$**.

Enter the fraction by using the fraction key,





To convert from a fraction to a decimal press the [**S↔D**] key.



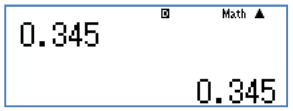
Non-Recurring Decimals

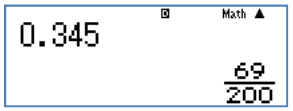
There are decimals that go on and on forever, but don't have any digits recurring in a pattern. The most common examples of a non-recurring decimal is **pi**, π which is **3.1415926535897932385…** or **e (Euler's number)** which are **2.718281828… or $\sqrt{2}$ = 1.414213562373095...**

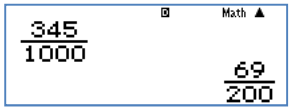
Fractions and Decimals

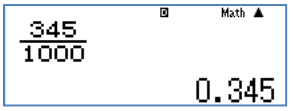
In decimal form, a rational number is either an exact or a recurring decimal. The opposite to this is also true, both exact and recurring decimals can be **always** be written as a fraction.

For example, $0.345 = \frac{345}{1000} = \frac{69}{200}$.







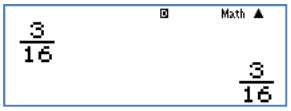


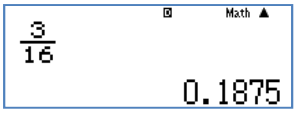
How can we tell if a fraction will be an exact or recurring decimal?

Fractions with denominators which the only prime factors are 2 and 5 will be exact decimals, any others will be recurring decimals.

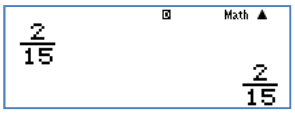
Are $\frac{3}{16}$ and $\frac{2}{15}$ exact or recurring decimals?

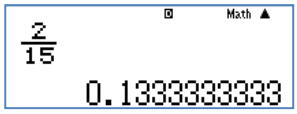
The denominator of $\frac{3}{16}$ is 16, which can be written as **$2 \times 2 \times 2 \times 2 = 2^4$**. The only prime factors of 16 are 2, so this is an exact decimal, (=0.1875).





Now look at $\frac{2}{15}$. If we break the denominator into it's prime factors we can see that **15 = 3 × 5**. Because there is a 3 in the prime factor decomposition then we know that $\frac{2}{15}$ is a recurring decimal, $0.1\dot{3}$ to be exact.





Converting a Recurring Decimal to a Fraction

We already know that every single recurring decimal can be written as a fraction. Most modern calculators can do this, by using algebra we can see how it's done.

Example 1:
Convert **0.4848484848…** into a fraction in it's simpliest form:
First let **x = 0.4848484848…**

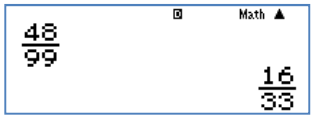
Now what we want to do is to move the first repeating section to the left hand side of the decimal point. With this example the repeated section is **..48..**, so multiplying **0.4848484848…** by 100 will achieve this.

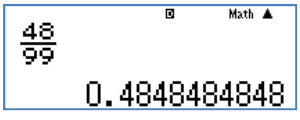
$100x = 48.4848484848\ldots$
$100x - x = 48.4848484848\ldots - 0.4848484848$
$99x = 48$

This is just now an equation to solve, divide both sides by 99 to get: **x =** $\frac{48}{99}$

Remember that the questions asked for the fraction in it's simplest form, so dividing the numerator and denominator by 3 gives:

0.4848484848… = $\frac{16}{33}$

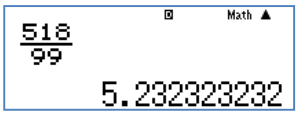


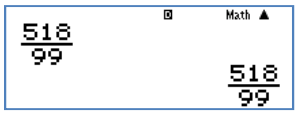


This method can be applied for **any** recurring decimal, not matter how long or complicated.

Example 2:

Convert 5.23232323… into a fraction.
Let $x = 5.23232323\ldots$
$100x = 523.232323\ldots$
$100x - x = 523.232323\ldots - 5.23232323\ldots$
$99x = 518$
$x = \frac{518}{99}$





Exercise 1

Are the following fractions exact or recurring decimals?

1. $\frac{1}{25}$
2. $\frac{1}{125}$
3. $\frac{9}{245}$
4. $\frac{1}{42}$
5. $\frac{135}{99}$

Exercise 2

Convert the following to decimals into fractions.

1. 0. 45454545…
2. 0.218218218…
3. 0.945945945…
4. 0.674567456745…
5. 1.1212121212…