

# Nth roots of a Complex Number - $z^n = (a + i b)$

This resource was written by Derek Smith with the support of CASIO New Zealand. It may be freely distributed but remains the intellectual property of the author and CASIO.

Select PRGM icon (press B) from the main menu or by using the arrow keys to highlight and then press EXE.



De Moivre's Theorem will be very helpful here.

$$z^n = (a + i b)^n = (r \cos \theta + i \sin \theta)^n = r^n (\cos n\theta + i \sin n\theta)$$

So to find the nth root of a complex number we get:

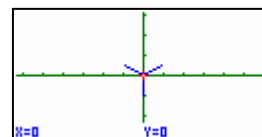
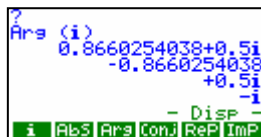
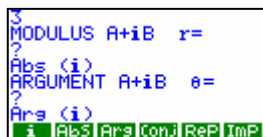
$$\begin{aligned} z &= (z^n)^{1/n} = ((a + i b)^n)^{1/n} \\ &= ((r \cos \theta + i \sin \theta)^n)^{1/n} \\ &= r^{n/n} (\cos (\frac{n\theta}{n} + \frac{2m\pi}{n}) + i \sin (\frac{n\theta}{n} + \frac{2m\pi}{n})) \end{aligned}$$

This can be written in shortened notation (polar form):  $z = r^{m/n} \text{cis}(\frac{m\theta}{n} + \frac{2m\pi}{n})$

## For example:

Solve  $z^3 = i$ , as  $i = 0 + 1i$ , the modulus of  $z^3$  is  $r = \sqrt{(0^2 + 1^2)} = 1$  and the argument is  $\pi/2$ , as  $\tan(\theta) = 1/0$ , implying  $\theta = \pi/2$ .

Yielding  $z = \text{cis}(\frac{\pi}{6} + \frac{2m\pi}{3})$ ,  $m = 0, 1, 2, 3, \dots$  **BUT**  $m = 0, 1, 2$  will reveal the 3 roots to  $z^3 = i$ , as when  $m = 3$ , this will be the same answer to  $m = 0$ , similarly  $m = 4$  gives the same answer when  $m = 1$  and so on.



Solutions:  $z_0 = 0.866 + 0.5i$      $z_1 = -0.866 + 0.5i$      $z_2 = -i$   
 $[z_0 = \sqrt[3]{1/2} + 0.5i$      $z_1 = -\sqrt[3]{1/2} + 0.5i$      $z_2 = -i]$   
 $[z_0 = \text{cis}(\pi/6)$      $z_1 = \text{cis}(5\pi/6)$      $z_2 = \text{cis}(9\pi/6)]$

Write the following program as a new file in **PRGM**.

**Filename:COMPLEX**

Deg

0→A~Z

"Z^M=A+B M ="

? → M

"MODULUS A+,B r ="

? → r

"ARGUMENT A+,B θ ="

? → θ

ViewWindow -6.3,6.3,1,-3.1,3.1,1

Lbl 1

1÷Mx(θ+2xNx180) → A

$r^{(1÷M)}(\cos A, \sin A)$  ◀

Plot 0,0

Plot  $r^{(1÷M)}\cos A, r^{(1÷M)}\sin A$

Line

N+1 → N

N=M+1=>Goto 2

Goto 1

Lbl 2

Plot 0,0

**Example:**

Solve  $z^3 = 8(1 + i)$

Run the program called “**COMPLEX**” via MAIN MENU ICON **PRGM**

Enter in 3 **EXE**  
This is the power.

```

Pros "COMPLEX"
Z^M=A+iB M=
?
3
    
```

Enter in the modulus [Abs]  
Using **OPTN** **F3** CPLX **F2**

```

Pros "COMPLEX"
Z^M=A+iB M=
?
3
MODULUS A+iB r=
?
Abs (8(1+i)
?
i |Abs|Arg|Conj|ReP|ImP
    
```

Enter in the argument [Arg]  
Using **OPTN** **F3** CPLX **F3**

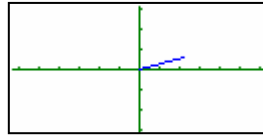
```

3
MODULUS A+iB r=
?
Abs (8(1+i)
ARGUMENT A+iB e=
?
Arg (8(1+i)
?
i |Abs|Arg|Conj|ReP|ImP
    
```

The ‘screensnaps’ that follow are shown below, after each **EXE** press:

```

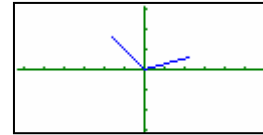
Abs (8(1+i)
ARGUMENT A+iB e=
?
Arg (8(1+i)
2.168430163
+0.581029111i
- Disp -
i |Abs|Arg|Conj|ReP|ImP
    
```



$z_0 = 2.168 + 0.581 i$

```

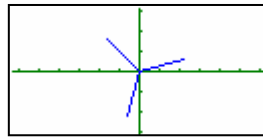
Arg (8(1+i)
2.168430163
+0.581029111i
-1.587401052
+1.587401052i
- Disp -
i |Abs|Arg|Conj|ReP|ImP
    
```



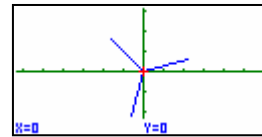
$z_1 = -1.587 + 1.587 i$

```

2.168430163
+0.581029111i
-1.587401052
+1.587401052i
-0.581029111i
-2.168430163i
- Disp -
i |Abs|Arg|Conj|ReP|ImP
    
```



$z_2 = -0.581 - 2.168 i$



The 3 roots of  $8(1 + i)$

**Points to note:** all the roots are symmetrically placed around the origin (0,0), i.e. the angle between each of the roots is  $120^\circ$ , all have the same modulus i.e.  $2\sqrt{2}$

Try solving

1.  $z^4 = 1$
2.  $z^3 = i$
3.  $z^6 = -1$
4.  $z^4 = 16(1 - i)$

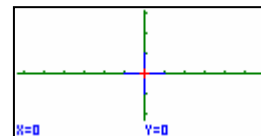
Answers:

```

4
MODULUS A+iB r=
?
1
ARGUMENT A+iB e=
?
0
    
```

```

?
0
- Disp -
    
```

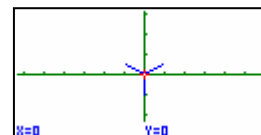


```

3
MODULUS A+iB r=
?
1
ARGUMENT A+iB e=
?
90
    
```

```

?
90
0.8660254038+0.5i
-0.8660254038
+0.5i
-0.8660254038
-0.5i
- Disp -
    
```



```

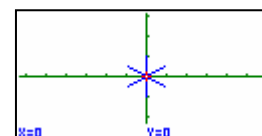
6
MODULUS A+iB r=
?
1
ARGUMENT A+iB e=
?
180
    
```

```

0.8660254038+0.5i
-0.8660254038
+0.5i
-0.8660254038
-0.5i
- Disp -
    
```

```

+0.5i
-0.8660254038
-0.5i
0.8660254038-0.5i
0.8660254038+0.5i
- Disp -
    
```



```

4
MODULUS A+iB r=
?
Abs (16(1-i))
ARGUMENT A+iB e=
?
Arg (16(1-i))
?
i |Abs|Arg|Conj|ReP|ImP
    
```

```

2.139107865
-0.4254950095i
-0.4254950095
+2.139107865i
-2.139107865
+0.4254950095i
- Disp -
    
```

```

-2.139107865
+0.4254950095i
-0.4254950095
-2.139107865i
-2.139107865
-0.4254950095i
- Disp -
    
```

