Finite and infinite series

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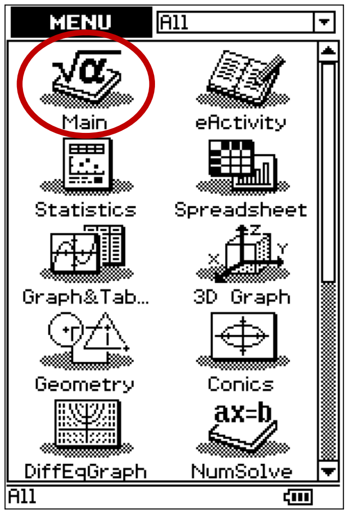


A sequence is a sequence of terms such as, <a_1, a_2, a_3, … >, this defines a sequence, where each term is connected by a mathematical rule. A finite sequence will end after a specified number of terms and an infinite sequence has an infinite number of terms, and has no ending term.
Such a series is represented (or denoted) by an expression like:
<2n + 1> = <3, 5, 7, 9, 11, … >

A series is an sum of a finite or infinite sequence of terms such as:
<a_1, a_1 + a_2, a_1 + a_2 + a_3, … >, this defines a series, where each term is connected by a mathematical rule. A finite series will end after a specified number of terms and an infinite series has an infinite number of terms, and has no end ding term.
Such a series is represented (or denoted) by an expression like:
<Σ 2n + 1> = <3, 3 + 5, 3 + 5 + 7, 3 + 5 + 7 + 9, 3 + 5 + 7 + 9 + 11, ….. >
= <3, 8, 15, 24, 35, … >
or, using the summation sign:

$$\sum_{\square=\square}^{\square}(\square)$$

For example:

$$\sum_{k=1}^{5}(2k+1)$$

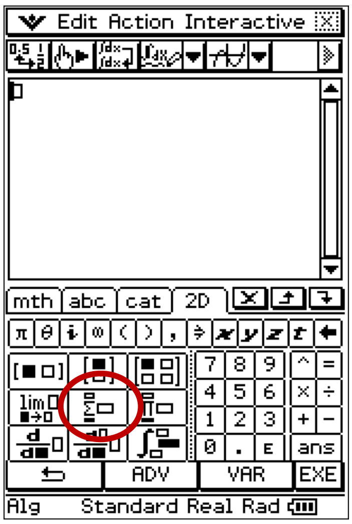
3 + 5 + 7 + 9 + 11 = 35
Finite series

$$\sum_{k=1}^{\infty}(2k+1)$$

3 + 5 + 7 + 9 + 11 + … = ∞
Infinite series

Enter the '**Main**' window from the **MENU**.
To bring up the sigma sign,:

1. Tap the '**2D**' tab.
2. Tap the '**CALC**' tab.
3. Σ is indicated below.

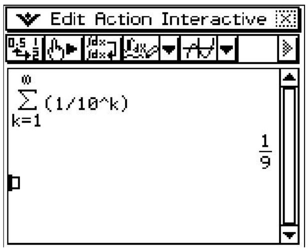


Now, all fractions can be represented as either a finite decimal or an infinite decimal (repeating decimal).
For example, the finite decimal 0.5 is the fraction ½, and the infinite, repeating decimal $0.\overline{3}$ = 0.333333… = $^1/_3$.

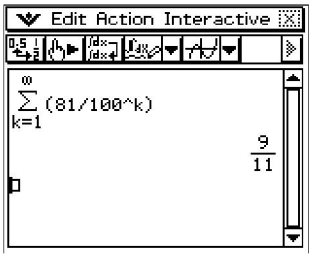
Other ways of expressing repeating decimals:

So, what is the fractional equivalent of $0.\overline{1}$?

$0.\overline{1} = 0.1 + 0.01 + 0.001 + 0.0001 + \ldots\ldots = \sum_{k=1}^{\infty}(1/10\text{^}k) = {}^1/_9$



$0.\overline{81} = 0.81 + 0.0080 + 0.000081 + 0.00000081 + \ldots = \sum_{k=1}^{\infty}(81/100\text{^}k) = {}^9/_{11}$



Try these:

$$1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\frac{1}{16}+\cdots=\sum_{n=0}^{\infty}\frac{1}{2^n}.$$

$$-1+\frac{1}{3}-\frac{1}{5}+\frac{1}{7}-\frac{1}{9}+\cdots=\sum_{n=1}^{\infty}\frac{(-1)^n}{2n-1}=-\frac{\pi}{4}$$

$$1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\frac{1}{5}-\cdots=\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{n}=\ln(2)$$

$$1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\cdots=\sum_{n=1}^{\infty}\frac{1}{n}.$$

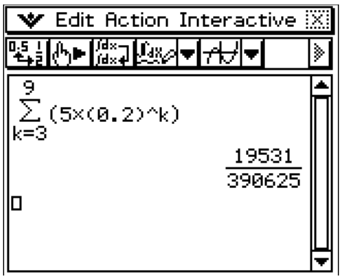
$$3+\frac{5}{2}+\frac{7}{4}+\frac{9}{8}+\frac{11}{16}+\cdots=\sum_{n=0}^{\infty}\frac{(3+2n)}{2^n}.$$

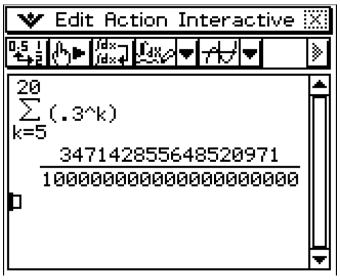
Some finite series.

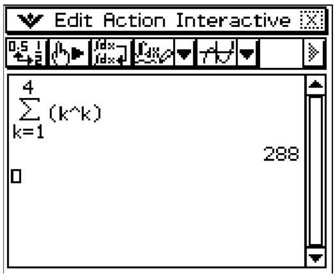
$5(0.2)^3 + 5(0.2)^4 + \ldots + 5(0.2)^9$

$$\sum_{n=5}^{20}(.3)^n$$

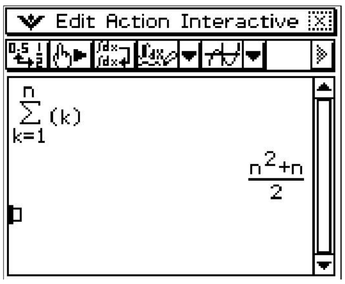
$$\sum_{k=1}^{4}(k\text{^}k)$$



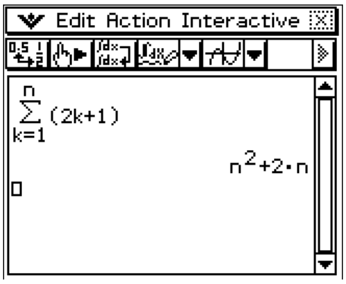




$< n > = 1 + 2 + 3 + 4 + \ldots + n$



$< 2n + 1 > = 3 + 5 + 7 + 9 + \ldots + n$



$< n^2 > = 1^2 + 2^2 + 3^2 + 4^2 + \ldots + n^2$