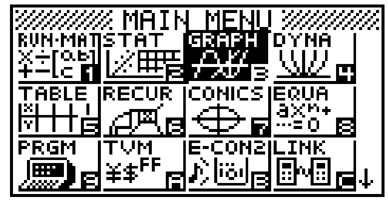
Linear Programming

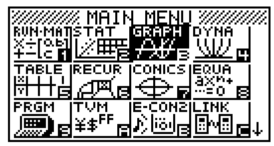
This resource was written by Derek Smith with the support of CASIO New Zealand. It may be freely distributed but remains the intellectual property of the author and CASIO.

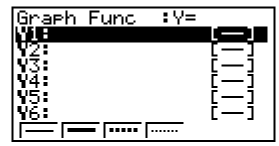
Linear Programming is a method to achieve the best outcome (such as maximum profit or lowest cost) in a mathematical model whose requirements are represented by linear relationships. A technique for the optimization of an objective function, subject to linear equality and/or linear inequality constraints. The feasible region is a 2-dimensional region, which is a set defined by the intersections of finite points (vertices), each of which is defined by the intersecting linear inequalities. The objective function is defined on this region. Linear programming finds a point (or series of points) in the region where this function has the smallest (or largest) which minimises (or maximizes) the objective function.



A **linear function to be maximized:**

$$f(x_1, x_2) = c_1x_1 + c_2x_2$$

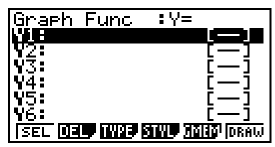




STYL [F4] Changing the style of the line drawn.

Problem constraints of the following form:

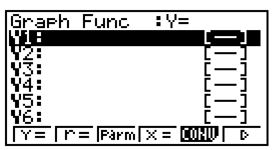
$$\begin{aligned} a_{11}x_1 + a_{12}x_2 &\le b_1 \\ a_{21}x_1 + a_{22}x_2 &\le b_2 \\ a_{31}x_1 + a_{32}x_2 &\le b_3 \end{aligned}$$



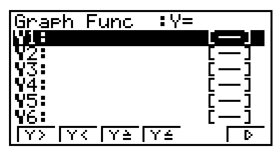
TYPE [F3]

Non-negative variables:

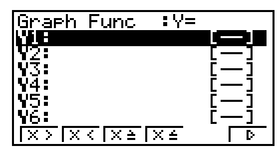
$$x_1 \ge 0$$
$$x_2 \ge 0$$



MORE CHOICES [F6]



[F1] [F2] [F3] [F4] … [F6]
Y> Y< Y ≥ Y ≤



[F1] [F2] [F3] [F4]
X> X< X ≥ X ≤

<u>Example:</u>
A farmer wishes to put in no more that 100 hectares of land into two types of crops, peas and beans. The pea crop requires twice as many bags as the bean crop per hectare. The farmer is able to afford no more than 160 bags in total. Other expenses include $3 000 per hectare of peas and $1 000 per hectare for the bean crop. The farmer cannot afford more than $24 000 in total, in expenses. The farmer estimates that the pea crop will return $6 000 per hectare and the bean crop return is $8 000 per hectare. How should the famer plant the land to maximize the return from the two crops.

Solution: Setting up the constraints.

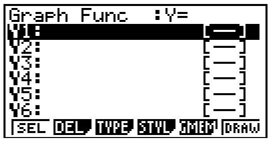
	Land (Hectares)	**Bags (Count)**	**Expenses ($)**
Pea crop	1	1	3 000
Bean crop	1	2	1 000
Constraints	≤ 100	≤ 160	≤ 240 000
Equations	x + y ≤ 100	x + 2y ≤ 160	3 000x + 1 000y ≤ 240 000

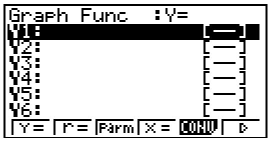
Together with x ≥ 0 and y ≥ 0

To maximize the profit equation: f(x, y) = 6 000x + 8 000y

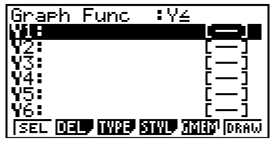
x + y ≤ 100	becomes y ≤ 100 - x
x + 2y ≤ 160	becomes y ≤ (160 – x) ÷ 2
3 000x + 1 000y ≤ 240 000	becomes y ≤ (240 000 - 3 000x) ÷ 1 000

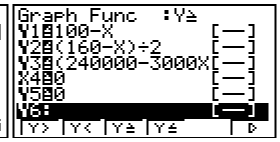
Enter the equations:



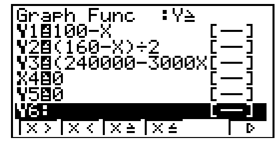


[F3] for Type

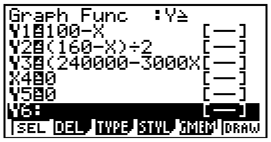




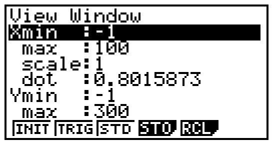
[F6] for Y inequalities



[F6] again for X inequalities

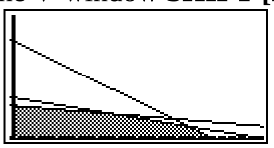


Set up the V-window **SHIFT [F3]**:

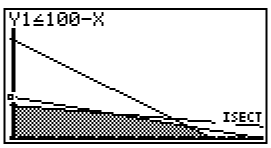


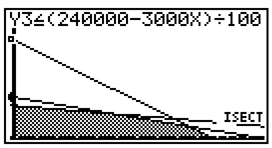
then **[EXIT]** followed by **[F6]** or **[EXE]** to draw.

Result:

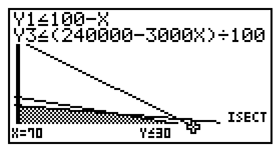


Now, using **G-Solv**e [**SHIFT**] [**F5**] and to find the intersection points [**F5**] for **ISCT** (Intersection). Selecting two equations at a time, select one then **[EXE]**, then the other followed by **[EXE]**.

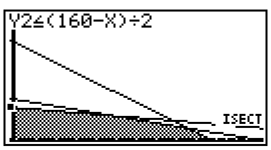


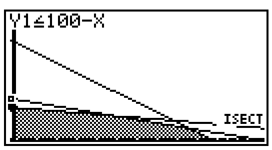


Result:

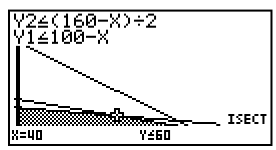


(70 , 30)

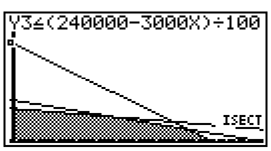


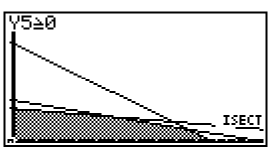


Result:

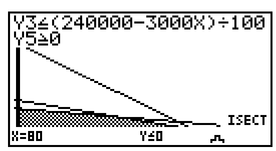


(40 , 60)





Result:



(80 , 0)

[Note: The calculator cannot find the point of intersections for vertical line x = c.]

Maximise the equation is f(x, y) = 6 000x + 8 000y.

x	y	f(x, y) = 6 000x + 8 000y.	Profit ($)
70	30	6 000×70 + 8 000×30 = 660 000	660 000
40	**60**	**6 000×40 + 8 000×60 = 720 000**	**720 000**
80	0	6 000×80 + 8 000×0 = 480 000	480 000
0	80	6 000×0 + 8 000×80 = 640 000	640 000

Maximum profit occurs for the farmer when there are 40 hectare of pea crops and 60 hectares of Bean crops planted.