What is the area of the largest rectangle ...

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What is the maximum area of a rectangle that can be placed inside another with the vertices of the inner rectangle on the sides of the outer rectangle?

Let's start with a square as the outer rectangle and animate an inner rectangle, where the vertices of the inner rectangle travel along the sides of the outer square and use the animation to capture the lengths and areas of the inner rectangle as the vertices move.

Enter the **Geometry** icon from **Main** and clear any pictures, diagrams or text, to start with a clear working area. Tap with the stylus on '**Edit**' and then '**Clear All**' then follow the onscreen instructions. Select '**View**' then place a '√' inside the box for '**Integer Grid**'. Adjust the screen so you can see the grid via '**View Window**' (see below) then tap '**OK**' to be displayed.
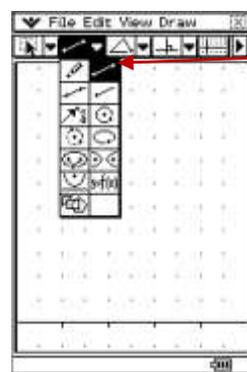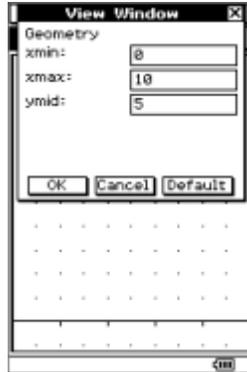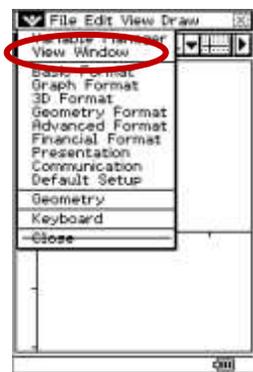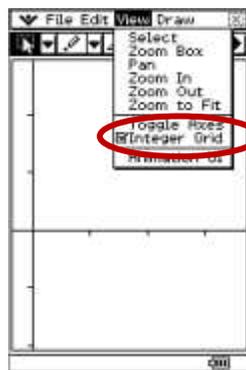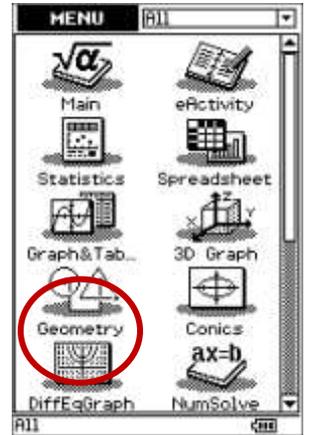

Line segment tool.

Create a square with sides of length 4 units. Create the line segment AB, then AC, then DB and DC, using the '**line segment tool**'.

[Note: the order is important so that the animations of the inner rectangle move in the appropriate directions.]

Next, using the '**point**' tool, tap on each of the sides of the square □ABCD to create the points E, F, G and H. Now connect E, F, G and H with line segments to form a quadrilateral.
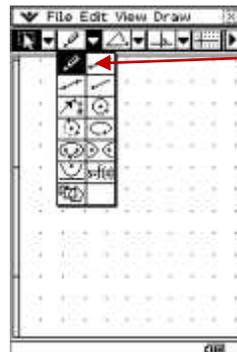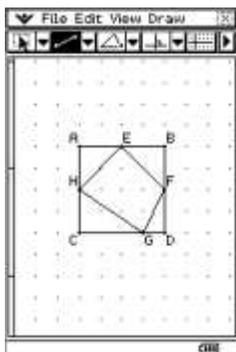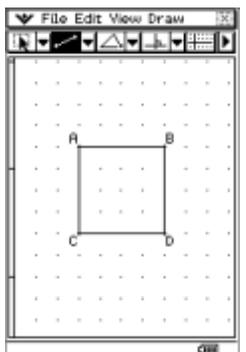
Point tool.

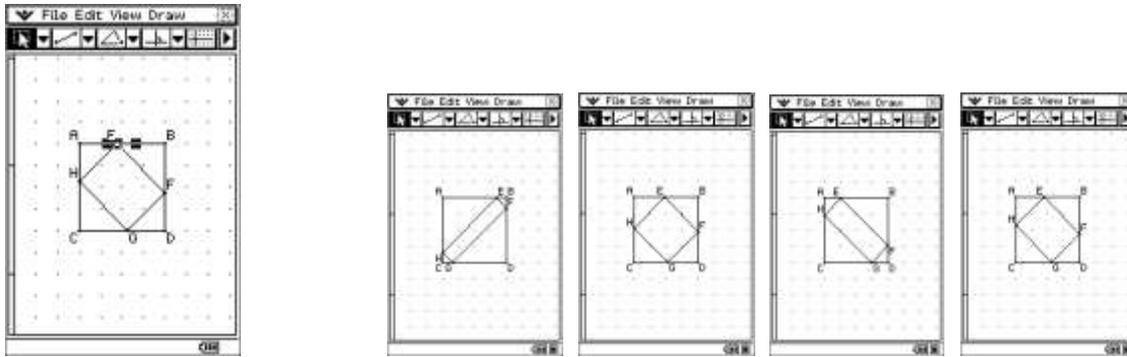Now animate the points E, F, g and G with their respective sides of the outer square. You need to animate each point with its respective side one at a time. For example, tap on point **E** then the line **AB**, then '**Edit**' then '**Animate**' and lastly tap on '**Add Animation**'.


Repeat this process to animate the other 3 points and their respective sides of the outer square. Now animate by selecting '**Edit**' then '**Go once**'. See some screen captures are illustrated above.

Select a side (e.g. **EF**) and then tap on the ▦ icon to display the captured results of the lengths of **EF** during the animation process. Select the 4 vertices of the inner parallelogram (**E**, **F**, **G** and **H**) and then tap on the ▦ icon again to display the captured results of the areas produced in the animation process.
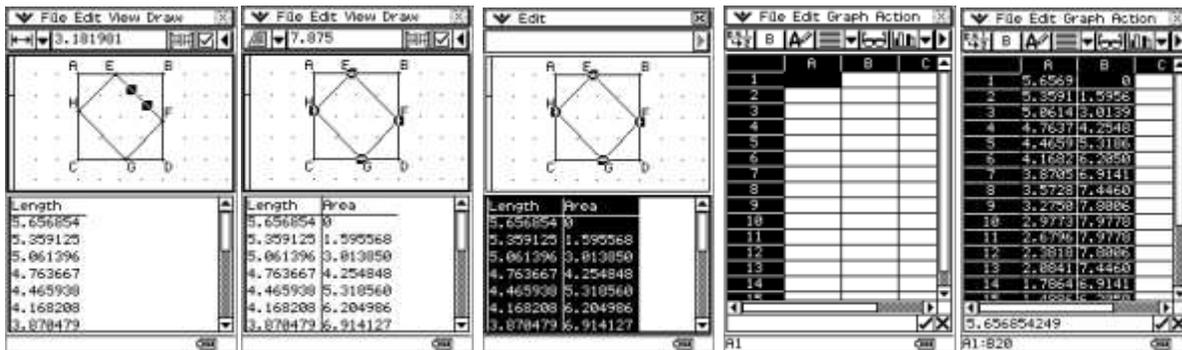
Select all of the data by highlighting the Length and Area columns, tap on '**Edit**', then '**Copy**' (See above). Tap on the **Main** icon (Bottom left area of the screen) and select the **Spreadsheet** icon, then '**Edit**' then '**Paste**' to transfer the animation data to the spreadsheet cells.
Now, create a scattergraph for Length versus Area.
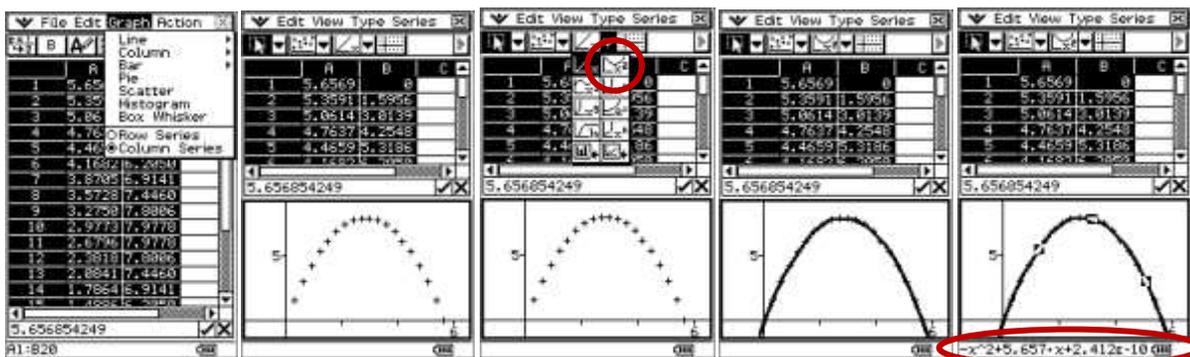
Select '**Graph**' then '**Scatter**'.   Select the mathematical model.   Mathematical model equation displayed.

Questions:
1. What is the length of the side **EF** (or **EH**), where the maximum area for the inner rectangle occurs?
2. How do you know that the inner rectangle **is** a rectangle?
3. What is the relationship of the side **EF** (or **EH**) to the diagonal of the outer square?

Variation:
What if the outer rectangle is NOT a square?
[Note: This will make the inner 4-sided figure is a parallelogram.] How do toy know this?

The example illustrated for this variation is a rectangle ABCD with sides of lengths 5 mu by 4 mu respectively. The construction is repeating the tapping process illustrated in the square example. Can you generalize for a $m \times n$ rectangle?
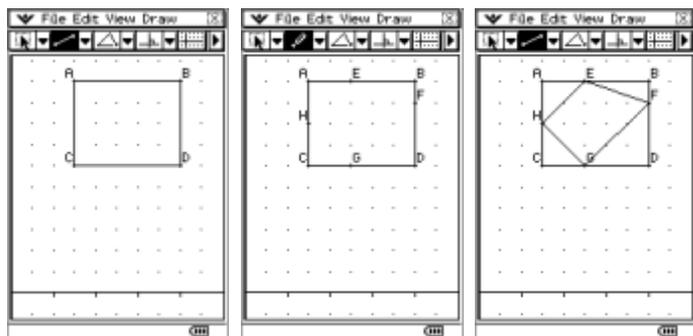

Now animate the points **E**, **F**, **G** and **H** with their respective sides of the outer square. You need to animate each point with its respective side one at a time. Tap on the point **E** then the line **AB**, then '**Edit**' then '**Animate**' and lastly '**Add Animation**'.
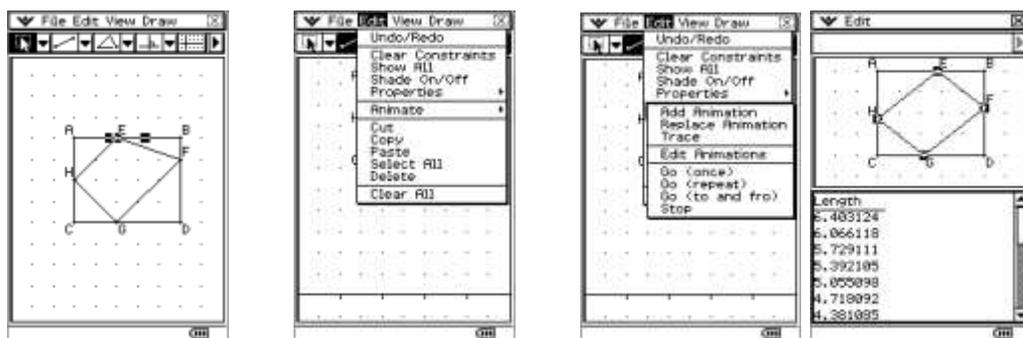
Repeat this for the other 3 points and lines. Now animate by selecting '**Edit**' then '**Go once**'. Some screen captures are illustrated below.
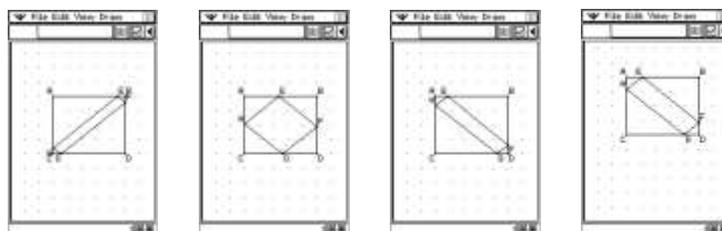
Select a side (e.g. EF) and then tap on the [⊞] icon to display the captured results of the lengths for the animation. Also select the 4 vertices of the inner parallelogram (E, F, G and H) and then tap on the [⊞] icon again to display the captured results of the areas for the animation.
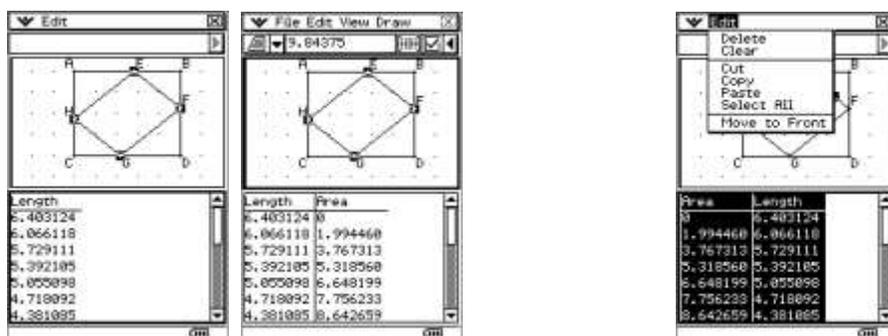
Select all of the data by highlighting the area and Length Columns, tap on '**Edit**' then '**Copy**' (See above).

Tap on the **Main** icon (Bottom left area of the screen) and select the **Spreadsheet** icon, then '**Edit**' then '**Paste**' to transfer the animation data to the spreadsheet cells.

Now, create a scattergraph for <u>Length</u> versus <u>Area</u>.
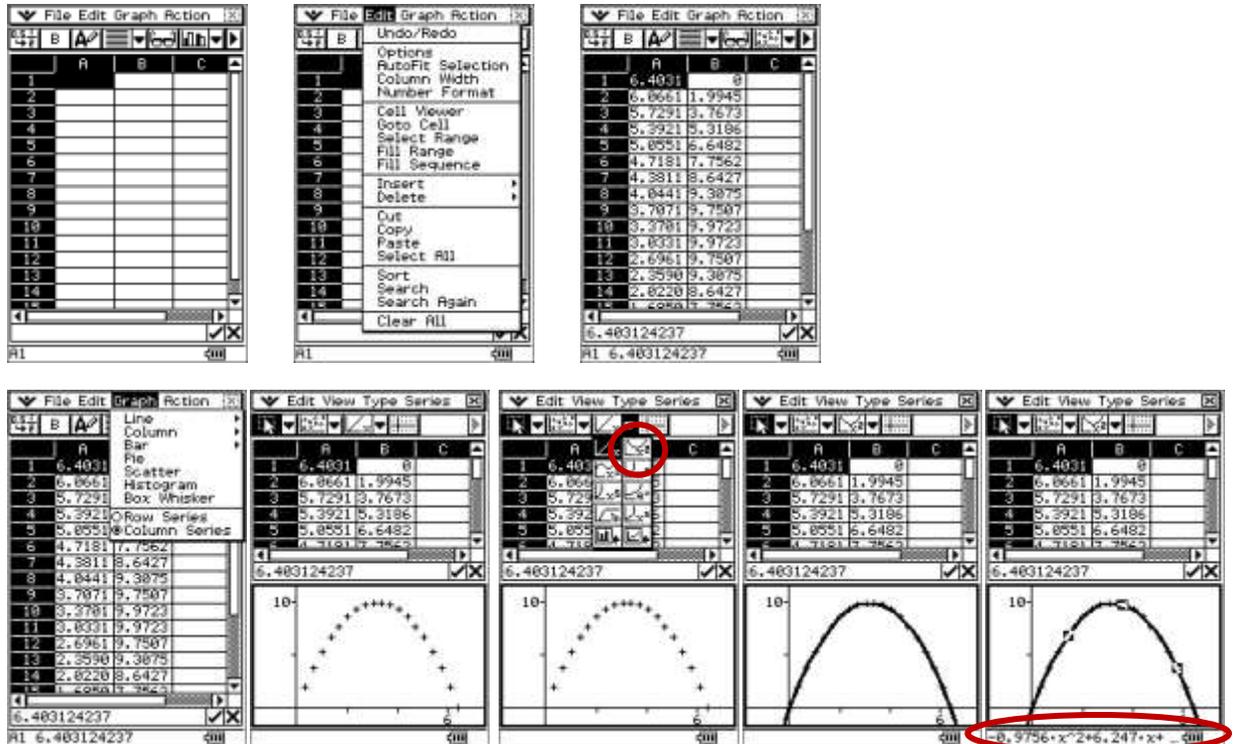
Select '**Graph**' then '**Scatter**'.   Select the mathematical model.   Mathematical model equation displayed.

Questions:
1. What is the length of the side **EF** (or **EH**), where the maximum area for the inner parallelogram occurs?
2. How do you know that the inner rectangle **is** a rectangle?
3. What is the relationship of the side **EF** (or **EH**) to the diagonal of the outer square?

Other variations:

Fit the largest quadrilateral inside a series of regular polygons with sides $n > 4$, …explore!

Questions:
1. What quadrilateral shapes is inside from the different polygons used as the outer?
2. What is the length of the sides of the quadrilateral inside the regular polygon, where the maximum area for the inner rectangle occurs?
3. How do you know that the inner rectangle **is** a quadrilateral?
4. What is the length of the inner quadrilaterals side relationship to the diagonal of the outer polygon?