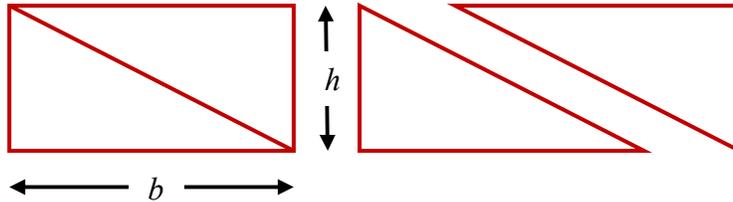


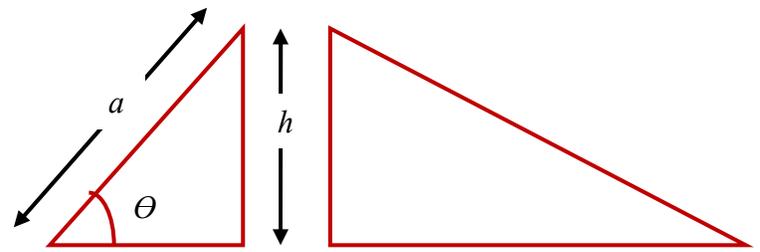
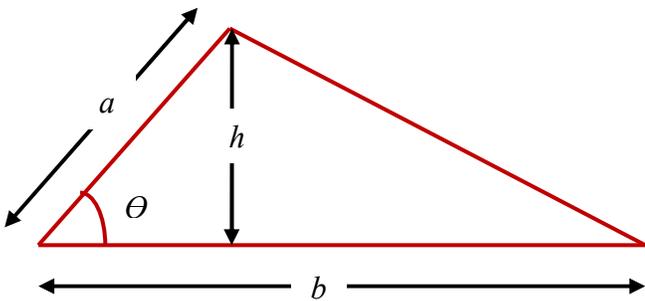
What is all this fuss about π ?

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The area of a right triangle is defined as $A = \frac{1}{2}b \times h$, where A represents the area, b the base length and h the height. This area formula can be derived from the area of a rectangle.



Extending this to a non-right angled triangle the area will be found using the same area formula.

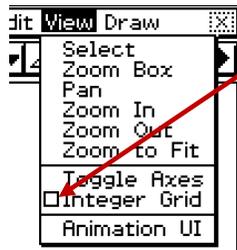
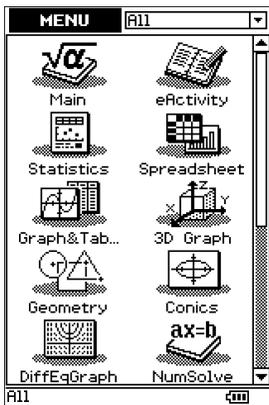


Using some trigonometry: $\sin \theta = \frac{h}{a}$

$$A = \frac{1}{2}b \times h = \frac{1}{2}a \times b \times \sin \theta = \frac{1}{2}ab \sin \theta$$

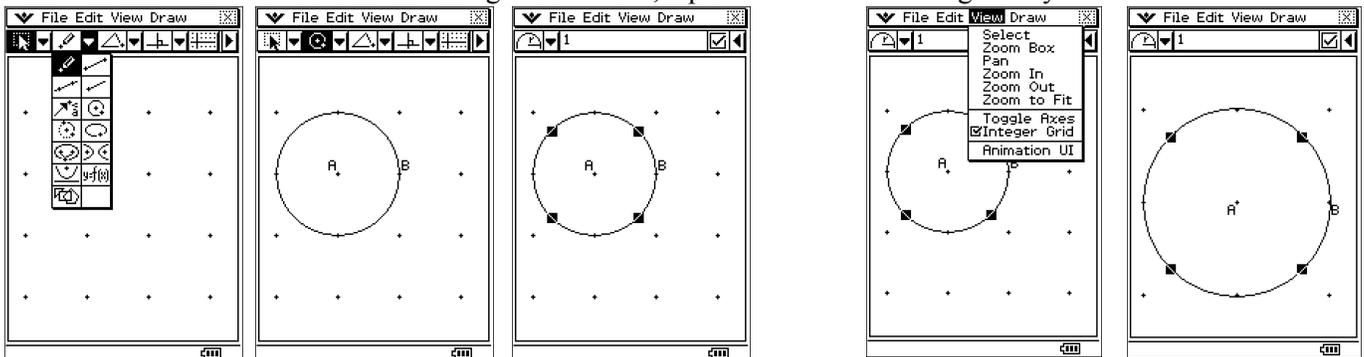
In the **Geometry** icon of the **ClassPad**.

Make sure that the integer grid and the x and y -axes is turned on by tapping on the **View** drop down box to place a tick in the 'Integer Grid' box.

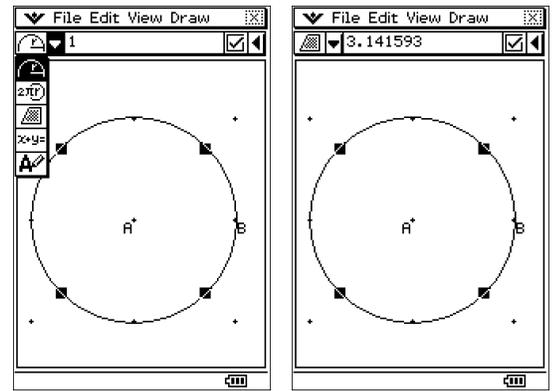


Tap to place a \checkmark in the box

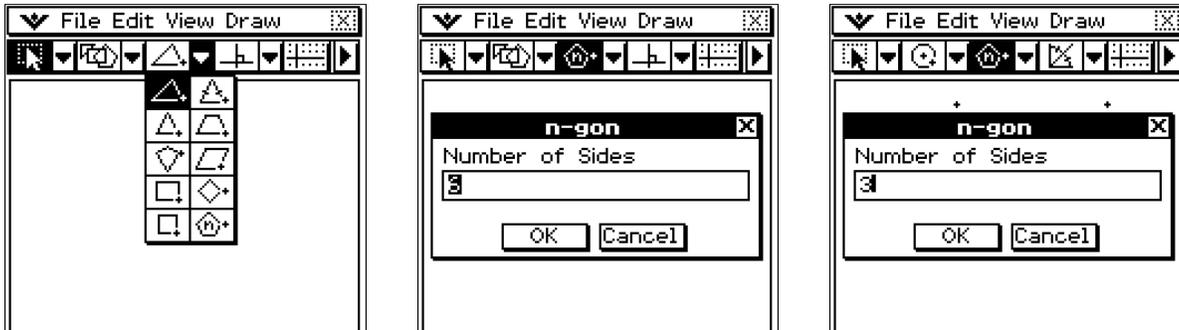
Draw a circle of radius 1 unit. You can draw a circle of any size and then constrain it to a radius of 1 unit length as shown below. Then resize the screen using 'Zoom Box', tap on the screen and drag the stylus to cover the circle.



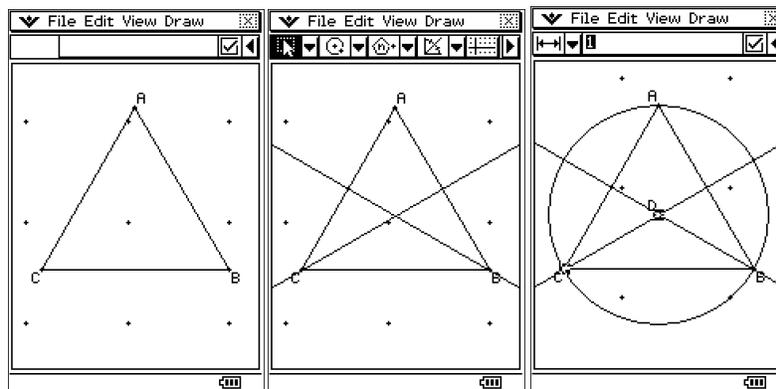
The area of this unit circle is given by $A = \pi r^2 = \pi \times 1^2 = \pi$. If you tap on the top left dropdown and select the area symbol you will see that the decimal equivalent (to 6 decimal places) will appear in the 'work area'.



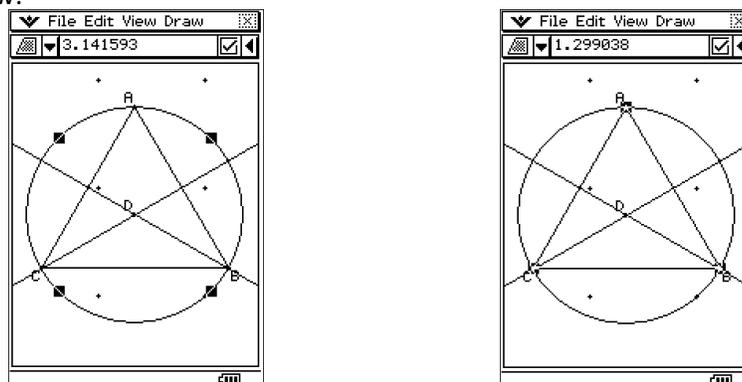
To construct a polygon select the drop down menu indicated in the screen capture below. You can change the number of sides for the polygon to be constructed by tapping in the work box and edit the number of sides required.



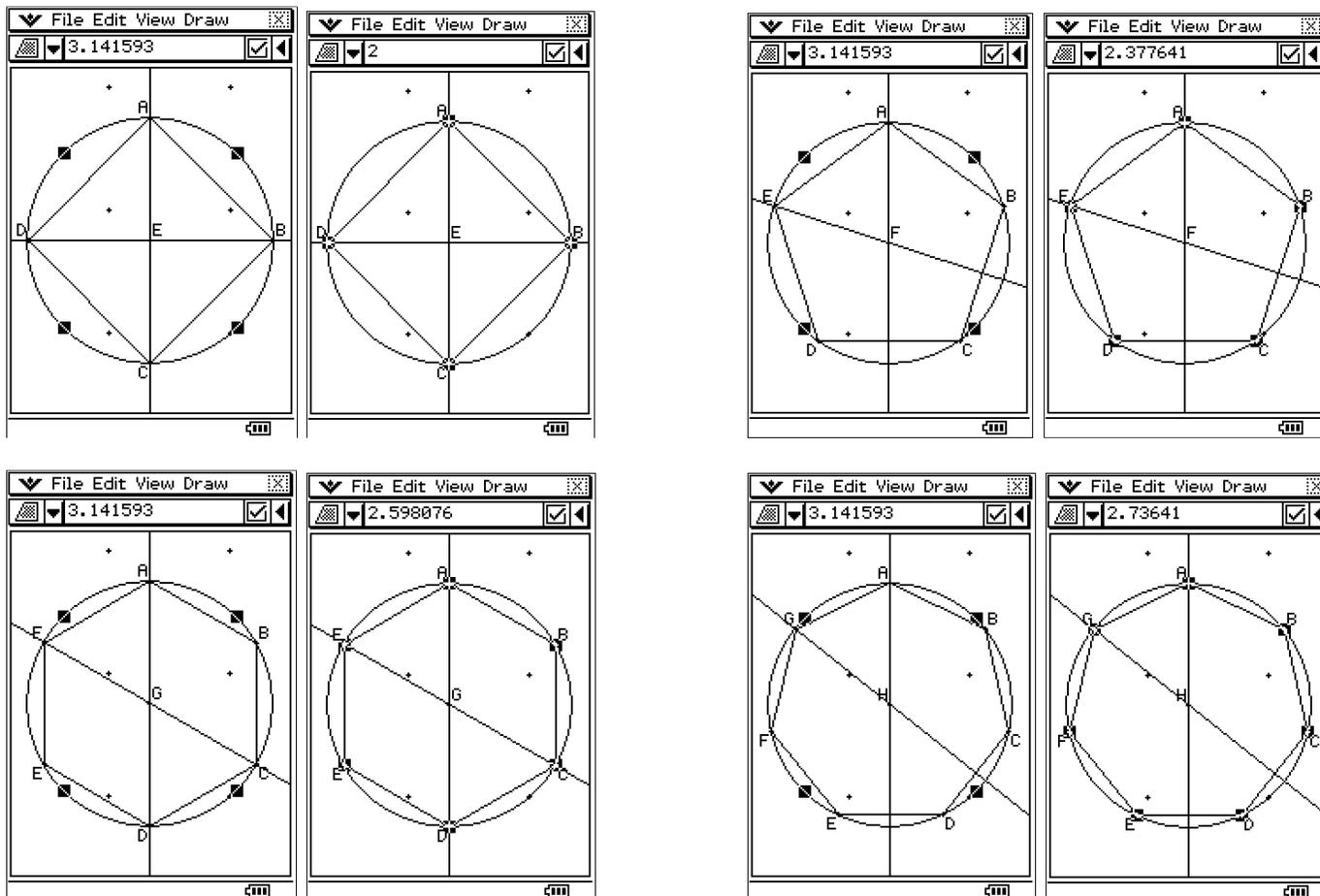
The default is 6-sides, change this value in the 'Number of Sides' box to 3. Construct an equilateral triangle and find its centre by bisecting two of the interior angles. Label the intersection point of these constructed bisecting lines and draw the circumcircle.



Resize the radius of the circumcircle by tapping on the points C and D and setting the length to be equal to 1. This will make the area of the circumcircle to be equal to π (3.141593) and the area of the equilateral triangle equal to 1.299038 μ^2 , as shown below.



Now, construct a series of polygons, $n = 4, 5, 6, 7, \dots$ and complete the same constructions as performed with the equilateral triangle.

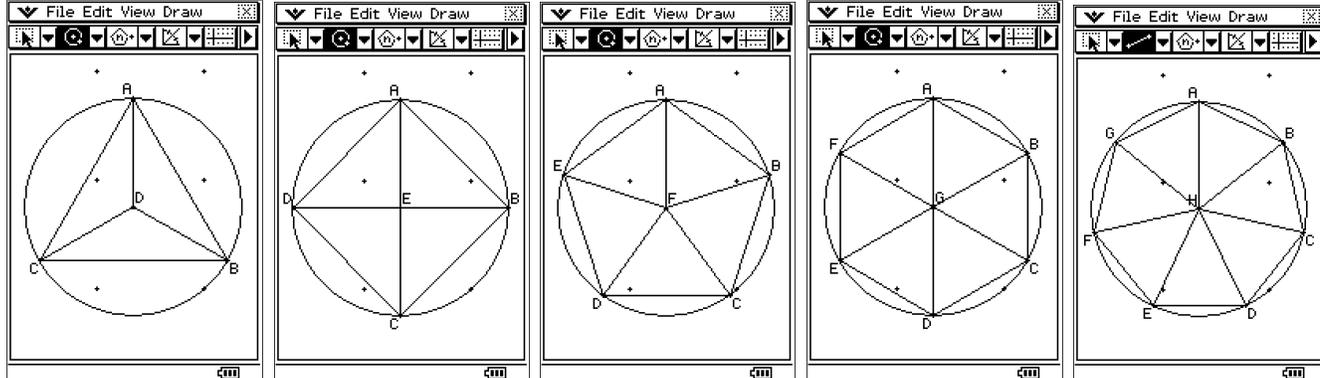


As we create polygons with an increasing number of sides the area of the polygon is getting larger BUT is constrained by the value π .

As the number of the sides of the polygon increases the area approaches π .

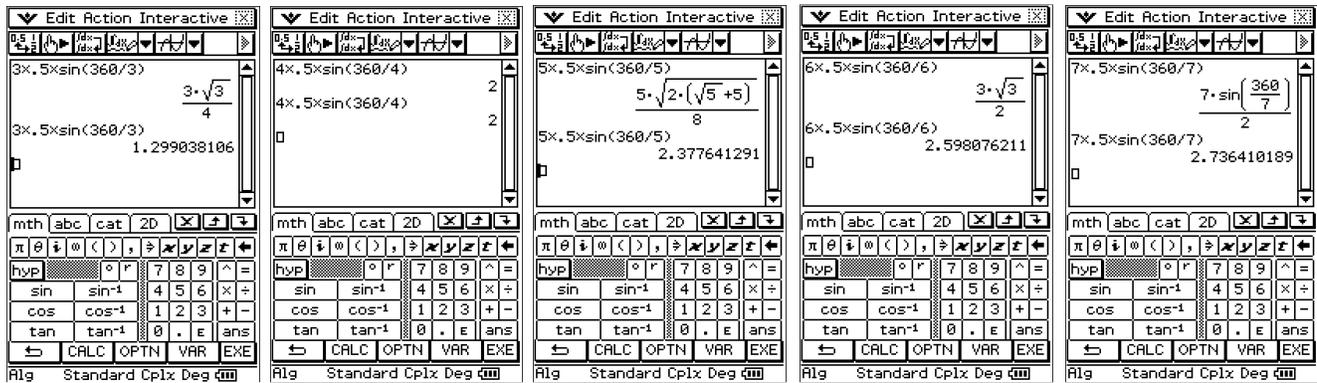
Using the area formula for a non-right angled triangle $A = \frac{1}{2}b \times h = \frac{1}{2} a \times b \times \sin \theta$ where the lengths of $a = b = 1$ gives the area of each triangle that can be constructed inside the polygon as $\frac{1}{2} n \sin \theta$

For an equilateral triangle, the number of triangles = 3, for a square = 4, for a pentagon = 5, for a hexagon = 6, ... for a n-gon = n.

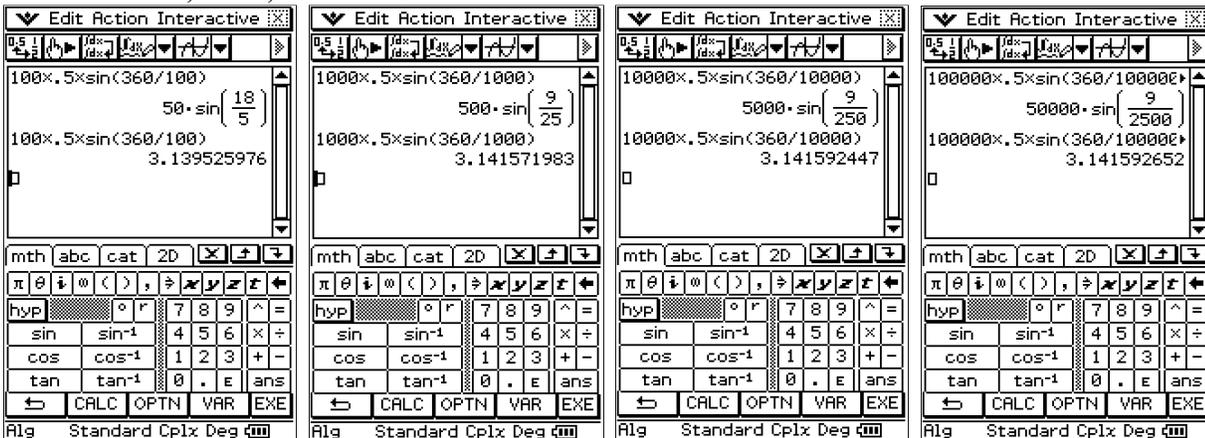


Polygon number of sides	3	4	5	6	7	8	9	10
Number of triangles	3	4	5	6	7	8	9	10
Area of each triangle	$\frac{1}{2}\sin(360^\circ/3)$	$\frac{1}{2}\sin(360^\circ/4)$	$\frac{1}{2}\sin(360^\circ/5)$	$\frac{1}{2}\sin(360^\circ/6)$	$\frac{1}{2}\sin(360^\circ/7)$	$\frac{1}{2}\sin(360^\circ/8)$	$\frac{1}{2}\sin(360^\circ/9)$	$\frac{1}{2}\sin(360^\circ/10)$
Total area of polygon	$3 \times \frac{1}{2}\sin(360^\circ/3)$	$4 \times \frac{1}{2}\sin(360^\circ/4)$	$5 \times \frac{1}{2}\sin(360^\circ/5)$	$6 \times \frac{1}{2}\sin(360^\circ/6)$	$7 \times \frac{1}{2}\sin(360^\circ/7)$	$8 \times \frac{1}{2}\sin(360^\circ/8)$	$9 \times \frac{1}{2}\sin(360^\circ/9)$	$10 \times \frac{1}{2}\sin(360^\circ/10)$

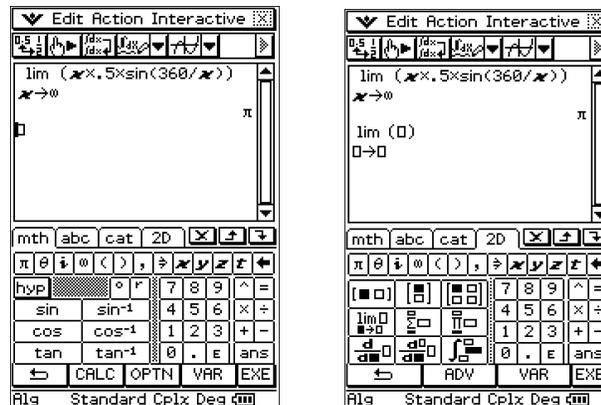
In the Main icon of the ClassPad.



Now make n = 100, 1 000, 10 000 and 100 000.



Using the 2D soft keypad for limits.



We can see that in the limit as $n \rightarrow \infty$, the area of the n-gon $\rightarrow \pi$.

$$\lim_{n \rightarrow \infty} n \times \frac{1}{2} \times \sin\left(\frac{360^\circ}{n}\right) = \pi$$