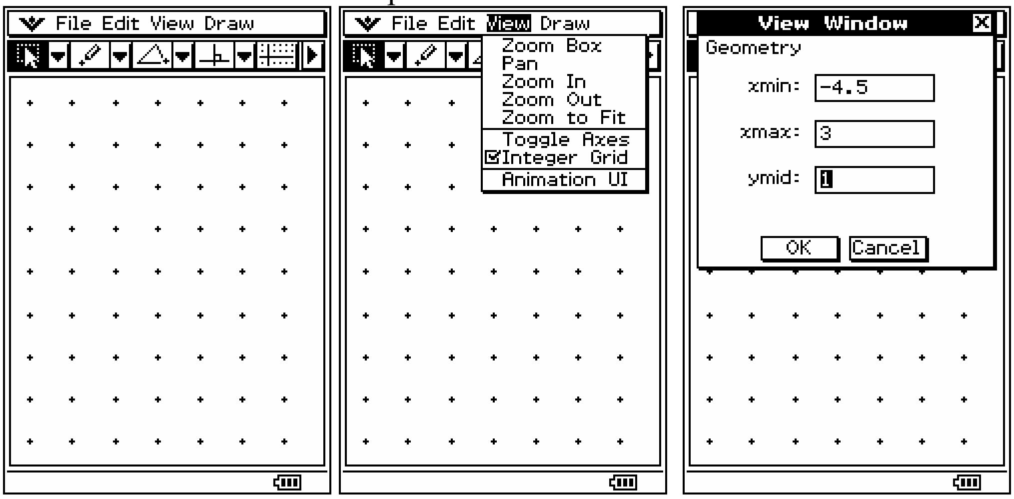
The length of chords of regular polygons in a unit circle.

This resource was written by Derek Smith with the support of CASIO New Zealand. It may be freely distributed but remains the intellectual property of the author and CASIO.

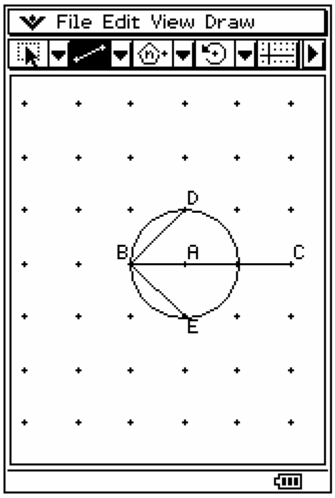
This is an investigation about the relationship between the lengths of the chords from a vertex of a regular polygon inscribed in a unit circle.

In the Geometry icon, set up the window so that the integer grid is 'toggled', zoom in so that you have a similar window illustrated or set up the View Window as illustrated below.

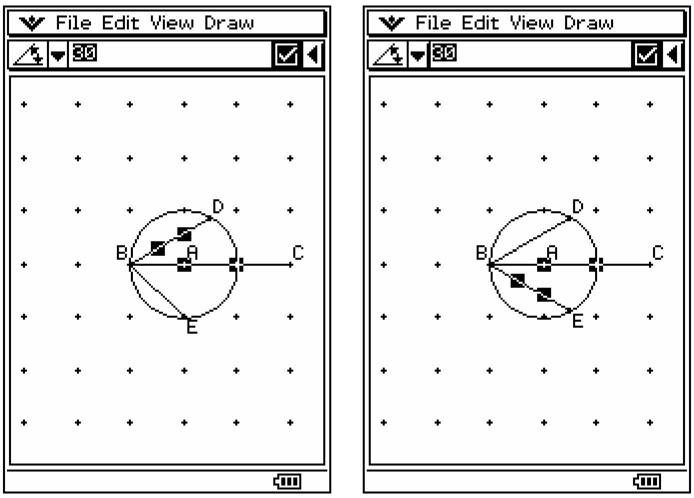


Create a circle of radius 1 unit and the from the point **B** draw a line segment that passes through the centre **A** and beyond. Now draw 2 chords from point **B**.

Tap on the line **BD** and **BC** and make **BD** have an angle or 30º to the line **BC**. You can do this by changing the angle in the working box to 30 and then tapping in the √ box, so it become shaded – this means that it is now a constraint that the angle 30º has been made to the line **BD** and **BC**. Repeat for **BC** and **BE**.

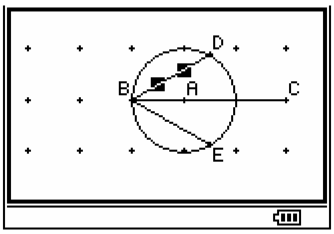


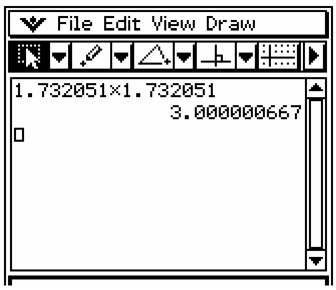
Construction stage.



Fixing the angle to 30º for **DBC** and **CBE**.

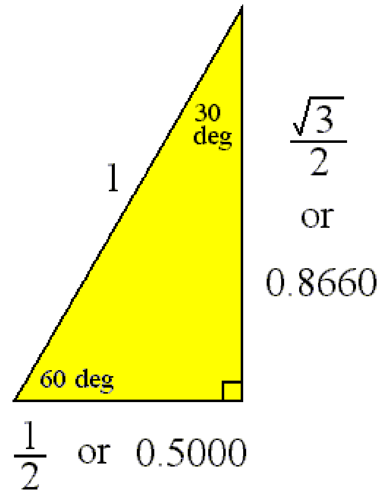
Now tap on the chord **BD** and determine its length. Similarly do the same for the chord **BE**.
Note: By construction **BD** = **BE**. What is the product of these two lengths **BD** x **BE**? You can highlight the length then copy and paste in the **MAIN** icon.

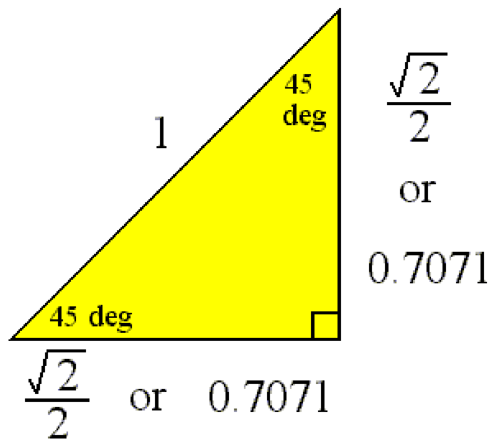




[N.B. You will notice that there is some rounding error creeping into the calculations.]

√3 x √3 = 3, as the triangle formed in the semicircle is a right angle and by construction it is a 30º-60º-90º triangle. Similarly for the square, a 45º-45º-90º triangle.



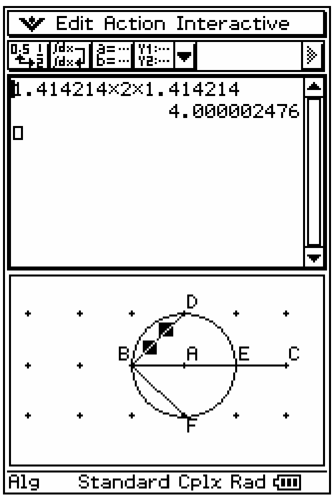


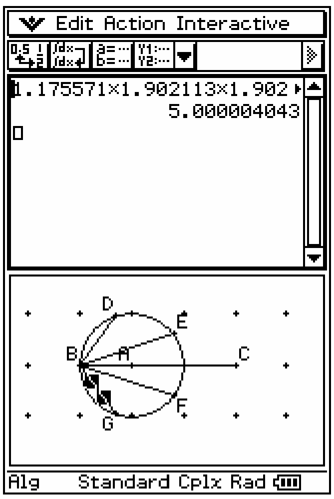
Repeat the process for constructing a square, pentagon and a hexagon inside a unit circle.

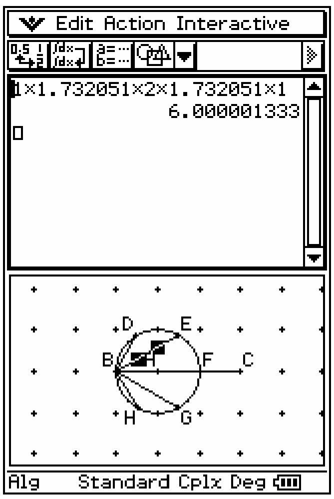
Shape	Chords from B	Angle construction with BC and other chord	Angle made with vertical line thru' B and the chord	Product of the chords
Equilateral triangle	BD and BE	60º, 60º	30º, 120º	3
Square	BD, BE and BF	45º, 0º , 45º	45º, 90º , 135º	4
Pentagon	BD, BE, BF and BG	54º, 18º , 18º, 54º	36º ,72º, 108º , 144º	5
Hexagon	BD, BE, BF, BG and BH	60º, 30º , 0º, 30º, 60º	30º ,60º, 90º , 120º, 150º	6

Calculations:

Equilateral triangle 2sin60ºx2sin120º = 3
Square 2sin45ºx2sin90ºx2sin135º = 4
Pentagon 2sin36ºx2sin72ºx2sin108ºx2sin144º = 5
Hexagon 2sin30ºx2sin60ºx2sin90ºx2sin120ºx2sin150º = 6



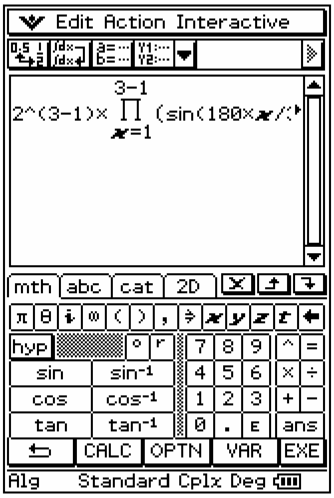


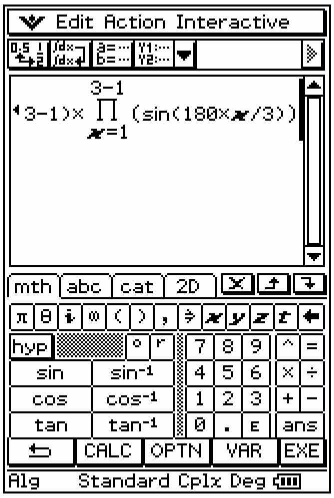


If we to generalise…

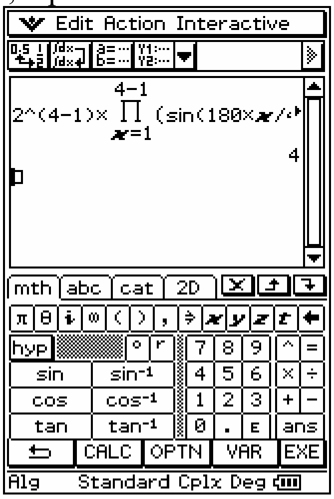
n-gon	n-1 chords with vertex B	…	$^{180i}/_{n}$ i = 1, 2,…,n-1	**Is it n?**

In **MAIN** use the 'soft key pad to enter in this expression.

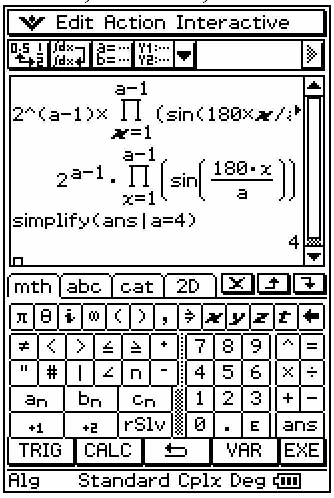




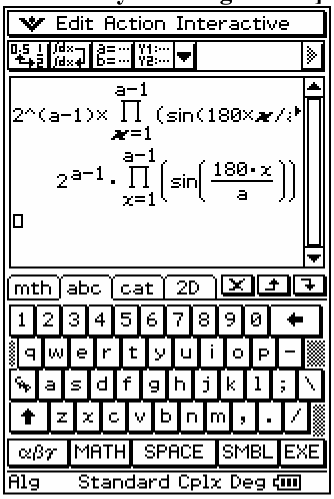
Now, replace the '3's' with 4's, then 5's, then 6's, then with 'a'. **[N.B. Make sure you change ALL!]**



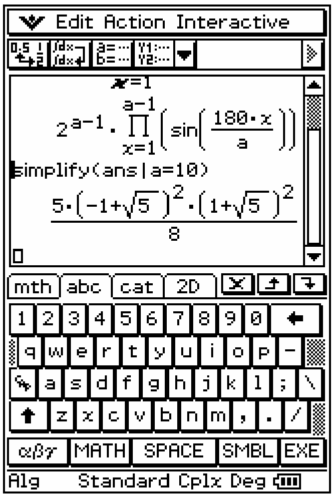
or

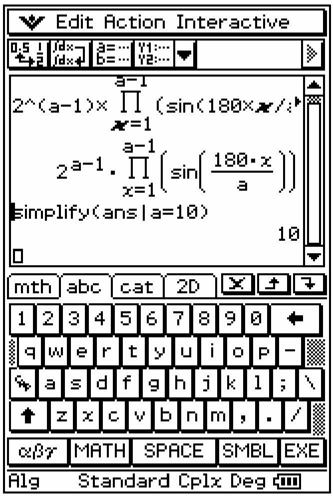


Now try,



What about 10?





What a great way to rewrite10!

Hence, $n = 2(n-1)\prod \sin({}^{180i}/_{n})$, $i = 1, 2,\ldots,n-1$ **Very elegant!**

$$2^{n-1}\cdot\prod_{x=1}^{n-1}\left(\sin\left(\frac{180\cdot x}{n}\right)\right)=n$$

Extension:

1. How could this proof be extended to a regular polygon inscribed in a circle of radius = r?
2. Prove $n = 2(n-1)\prod \sin({}^{180i}/_{n})$ $i = 1, 2,\ldots,n-1$ by mathematical Induction.