Series for finding Pi

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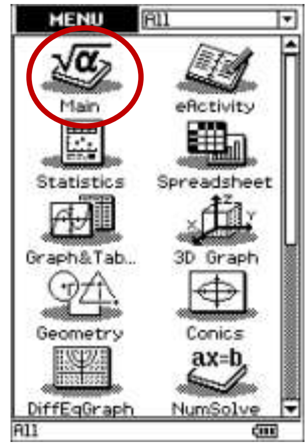


Pi (π), an irrational number has had mathematicians across time and around the globe a fascination about. Its discovery and relationship it has to many aspects of mathematics and probability are tributes to such an influential number!

Depending on the purpose of a calculation involving pi, it can be approximated by using fractions for ease of calculation. The most notable such approximations are $^{22}/_{7}$ and $^{355}/_{113}$. On the calendar there are two dates that are related to π and celebrated, namely:
22/7 (22nd July) and 3.14 (14th March).

$$\frac{22}{7} = 3.143^{+} \qquad \frac{355}{113} = 3.14159\,29^{+}$$

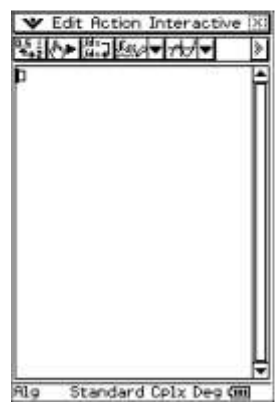
Her are some finite (terminating) examples of approximating π.

$$\sqrt{15} - \sqrt{3} + 1 = 3.140^{+} \qquad \sqrt{2} + \sqrt{3} = 3.146^{+} \qquad \sqrt[3]{31} = 3.1413^{+}$$

$$\frac{9}{5} + \sqrt{\frac{9}{5}} = 3.1416^{+} \qquad 3 + \frac{8}{60} + \frac{29}{60^2} + \frac{44}{60^3} = 3.14159\,259^{+} \qquad \sqrt{7 + \sqrt{6 + \sqrt{5}}} = 3.1416^{+}$$

$$\frac{7^7}{4^9} = 3.14156^{+} \qquad \pi \approx 768\sqrt{2 - \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + \sqrt{2 + 1}}}}}}}}} \approx 3.141590463236763.$$

Can you find others, test them, modify them to improve accuracy in approximating the value of π.



The use of a sequence (or series) of numbers, leading to algebraic expressions, have been used to approximate π over the course of time. Particularly Indian and Chinese mathematicians from about the 5th Century who used the concept of an infinite series.

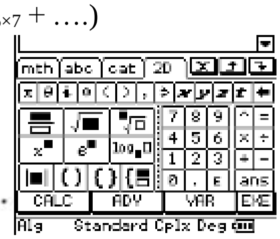
Use your ClassPad to experiment with some proven infinite series to estimate π. For instance, sin(30º) = sin($^{\pi}/_{6}$) = 0.5 (= ½), so rearranging gives: π = 6sin^{-1}(0.5).
Mathematicians discovered an infinite series for sin(x), if -1 ≤ x ≤ 1.
In the **Main** Icon bring up the soft **Keypad** and use the **2D** tab, **CAT** and **CALC**.

Example: $\sin(x) = {}^{x}/_{1!} + 1\times x^3/_{2\times3} + 1\times3\times x^5/_{2\times4\times5} + 1\times3\times5\times x^7/_{2\times4\times6\times7} + \ldots$

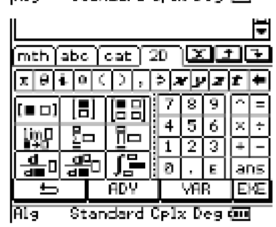
Giving $\pi = 6\sin^{-1}(0.5) = 6({}^{0.5}/_{1!} + 1\times0.5^3/_{2\times3} + 1\times3\times0.5^5/_{2\times4\times5} + 1\times3\times5\times0.5^7/_{2\times4\times6\times7} + \ldots)$

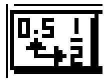
$$= \frac{3}{16^0 \cdot 1} + \frac{6}{16^1 \cdot 3} + \frac{18}{16^2 \cdot 5} + \frac{60}{16^3 \cdot 7} + \cdots = \sum_{n=0}^{\infty} \frac{3 \cdot \binom{2n}{n}}{16^n (2n+1)}$$

$$= 3 + \frac{1}{8} + \frac{9}{640} + \frac{15}{7168} + \frac{35}{98304} + \frac{189}{2883584} + \frac{693}{54525952} + \frac{429}{167772160} + \cdots$$



What value does your ClassPad give as an approximation for π?
How many terms can your ClassPad display until you get an 'Overflow' message?
Below is an example with 50 terms.



Tap the  icon to convert to a decimal.

Correct to 9 dp

Try these other infinite series to approximate π:

$$\pi = \sqrt{12}\sum_{k=0}^{\infty}\frac{(-3)^{-k}}{2k+1} = \sqrt{12}\sum_{k=0}^{\infty}\frac{(-\frac{1}{3})^k}{2k+1} = \sqrt{12}\left(1 - \frac{1}{3\cdot 3} + \frac{1}{5\cdot 3^2} - \frac{1}{7\cdot 3^3} + \cdots\right)$$

$$\frac{1}{\pi} = \frac{2\sqrt{2}}{9801}\sum_{k=0}^{\infty}\frac{(4k)!(1103+26390k)}{(k!)^4 396^{4k}}$$

$$\frac{1}{\pi} = 12\sum_{k=0}^{\infty}\frac{(-1)^k(6k)!(13591409+545140134k)}{(3k)!(k!)^3 640320^{3k+3/2}}$$

$$\frac{\pi}{2} = \sum_{k=0}^{\infty}\frac{k!}{(2k+1)!!} = \sum_{k=0}^{\infty}\frac{2^k k!^2}{(2k+1)!} = 1 + \frac{1}{3}\left(1 + \frac{2}{5}\left(1 + \frac{3}{7}\left(1+\cdots\right)\right)\right)$$

Source: https://en.wikipedia.org/wiki/Approximations_of_π