The Orthocentre.

Definition: In a triangle, an altitude is the line through a vertex perpendicular to the opposite side. The altitudes of a triangle intersect at a point called the orthocentre.

Tip: In the Geometry icon, set up the window so that the integer grid is 'toggled', zoom in so that you have a similar window illustrated or set up the View Window as illustrated on the right.
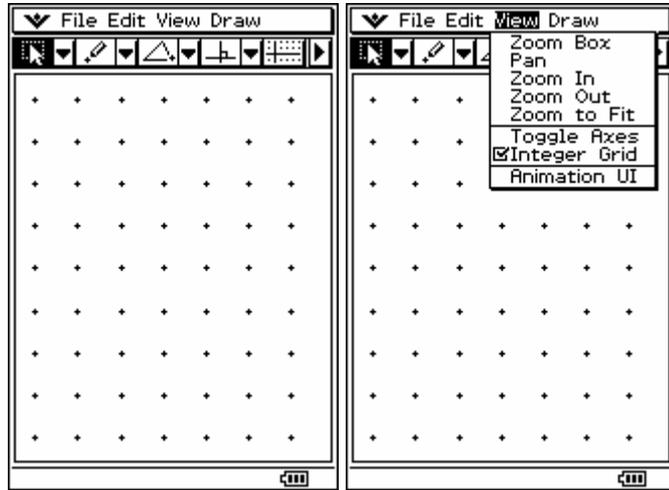
Question: The altitudes of a triangle intersect at a point called the orthocentre. Find the co-ordinates of the orthocentre, O of a triangle, Δ WXY, where the vertices of the triangle are W(-3 , -3), X(5 , -3) and Y(1 , 5).
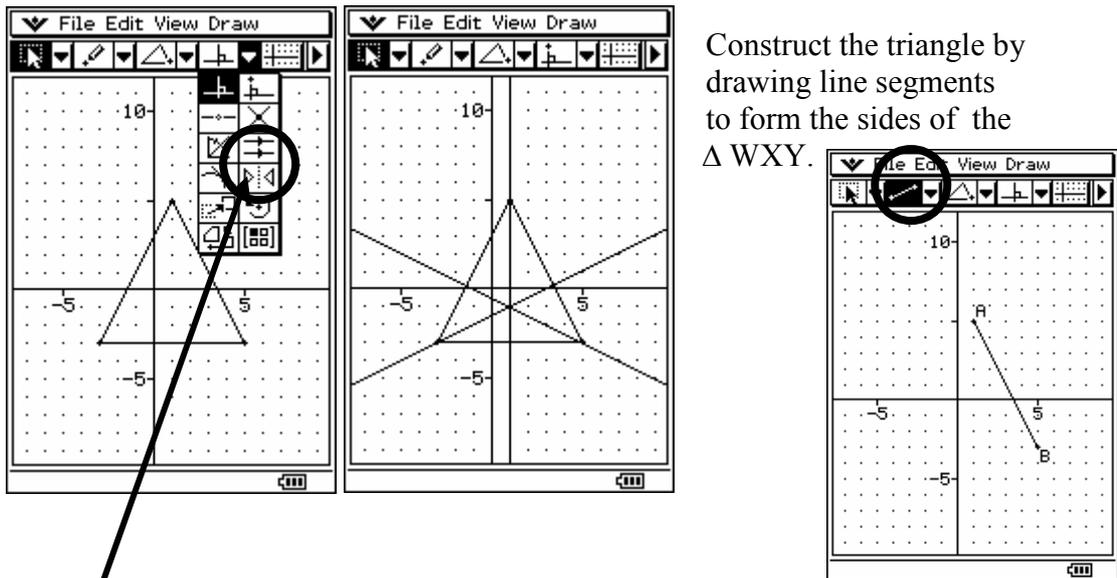

Construct the triangle by drawing line segments to form the sides of the Δ WXY.

Select the tool for constructing a perpendicular line through a given point [as indicated with the arrow]. Select the side of the triangle and the opposite vertex and then tap with the stylus onto the construct required. [N.B. Do each construction one at a time.]

Close the **Geometry** window and open the **Main.** Select the Geometry Icon as indicated with this arrow, this will open both windows so you can move between the two.

Working box
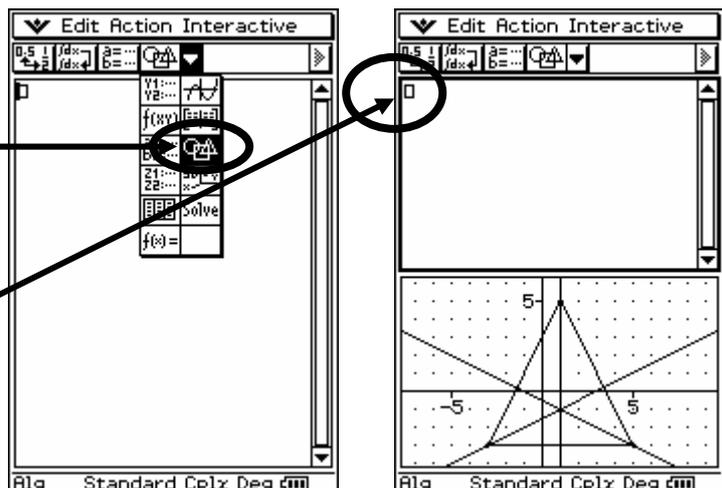
Tap on one of the **attitudes** to highlight it and then touch the altitude again with the stylus and drag, without taking the stylus off the screen, into the **Main** window and stopping to the left of the working box. [You might need to do this a few times to get the action correct – you will see a rectangle move as you drag over the geometry window and then see the cursor next to the working box if successful!]

The equation of that altitude should now be displayed in the Main window as shown in the screen snap on the right.
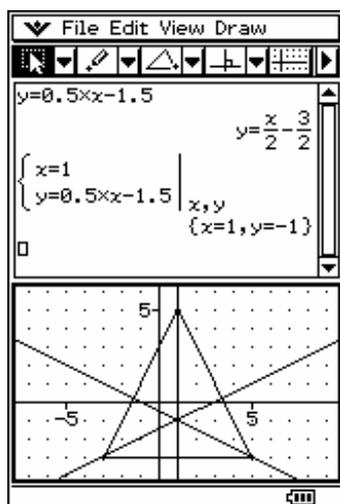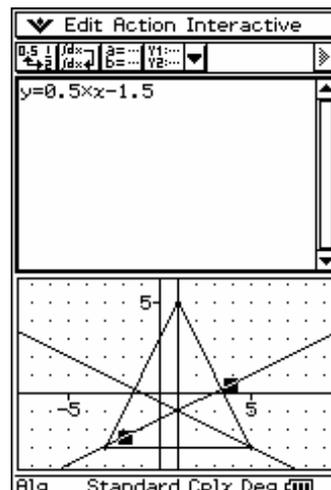
If you tap on two of the altitudes and drag them both into the into the main window and stopping to the left of the working box you have set up a simultaneous equation and pressing **EXE** will solve for (x,y) as indicated on the left screen snap.

You can repeat the process to find the equations of the another altitudes and solve simultaneously to find the intersection point, i.e. the orthocentre.

[N.B. you only need two altitudes, but it is good practice to find all three and solving simultaneously again will confirm the intersection point getting the same result.]

Hence the orthocentre has co-ordinates (1,1) for Δ WXY, with vertices W(-3,-3), X(5,-3) and Y(1,5).]

Extension:
With this construction of Δ WXY, with vertices W(-3,-3), X(5,-3) and Y(1,5) complete, change the co-ordinates of W, X and Y to W(0,3), X(5,3) and Y(2,5) and see what happens to the construction that you have made on the original Δ WXY. Carry out the same tapping and dragging sequence indicated above to find the new orthocentre, for the new Δ WXY.
[**Hint**: Tap the stylus to go to page 2 and type in the co-ordinates that you are changing to and tap in the '**tick box**'.]
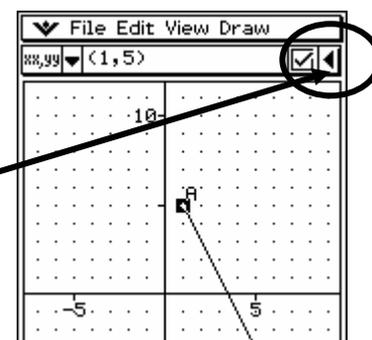
Using pencil and paper:
Find the gradients of the 3 sides of the triangle Δ WXY.
The equation of the altitude will be the negative reciprocal, [remember the condition for perpendicular lines, $m_1 \times m_2 = -1$] e.g. taking X(5,-3) and Y(1,5), the gradient of the line XY is $^{(-3-5)}/_{(1--3)} = -2$ so the gradient of the altitude will be ½ and hence its equation will be generated from the gradient = ½ and passing through the point (-3,-3) i.e. $y-y_1 = m(x-x_1)$ i.e. y--3 = ½(x--3)
y+3 = ½(x+3)
y = ½x-1½ Now, solve these two equations simultaneously…