Exploring the Limits of Functions

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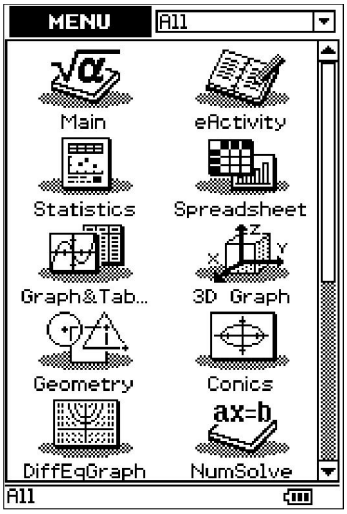


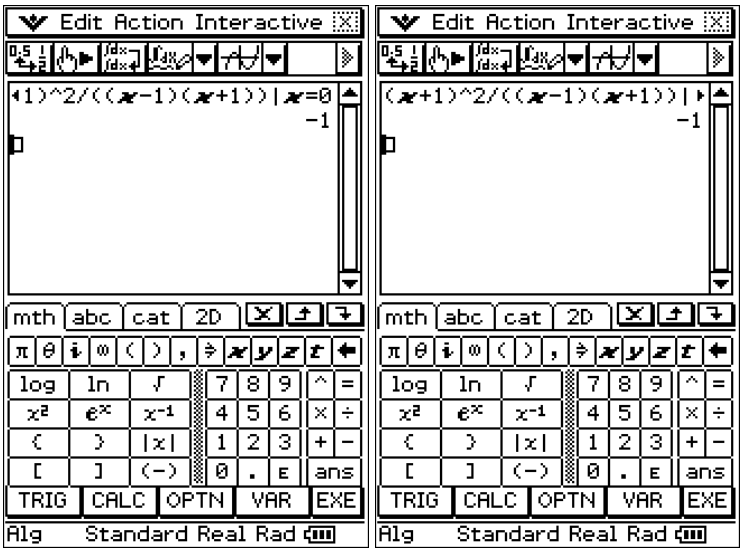
Definition: The **limit of a function** is one of the fundamental concept in calculus which concerns the behavior of the function near a particular value of x, on the domain. A function f assigns a value for $f(x)$ to every input x. We can say that a function has a limit at a point (p,q) so, as $f(x)$ gets closer and closer to q (the limit), as x moves closer and closer to p. this needs to be true as you approach the value of p from the left and right of p. If this is not the case then we can say the limit *does not exist*.

The use of limits extends the concept of substitution.

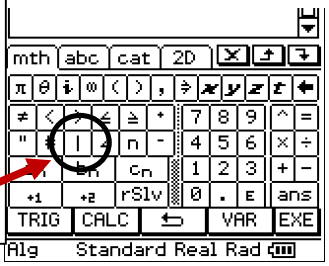
Exploring the use of the substitution notation on the ClassPad: '|'

Example: Calculate f(x), where $f(x) = \frac{(x+1)^2}{(x-1)(x+1)}$ when $x = 0$.

Solution: Substitution $x = 0$ into f(x) gives $f(0) = \frac{(0+1)^2}{(0-1)(0+1)} = \frac{1}{-1} = -1$

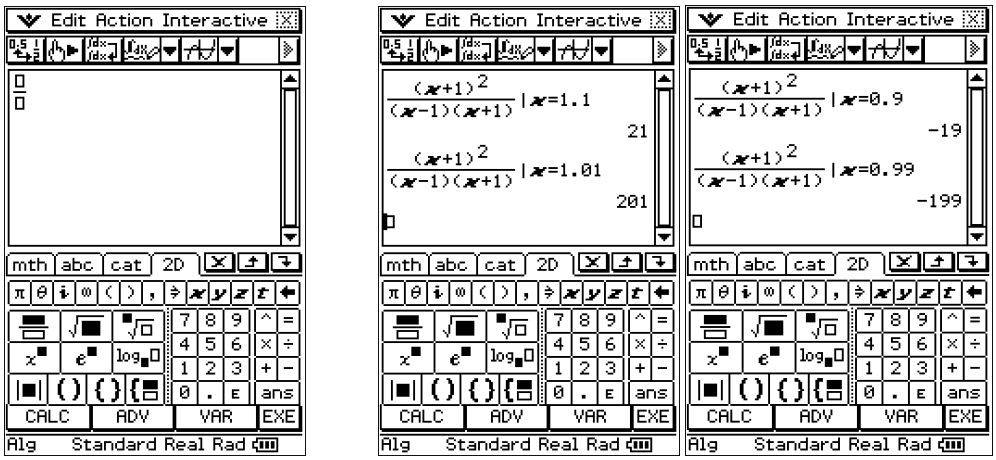


The | symbol can be located by opening the **Keyboard** and then tapping [**OPTN**] to open the 2nd set of terms. Then tap on [|]



Using the same f(x) there is an issue with f(x), when $x = 1$, this is because $f(1) = \frac{(1+1)^2}{(1-1)(1+1)} = \frac{4}{0} = ?$

So how can we determine if there is a limit as $x \to 1$?
We could look to see what happens by selecting values of x close to $x = 1$.
Note: This is demonstrated below using the **2D** keypad for the displaying f(x).

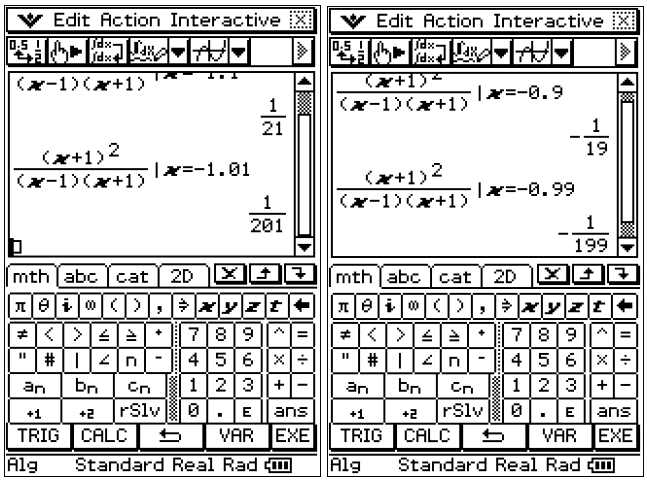


This shows that as the values of $x \to 1$. There is no limit.
As $x \to 1^-$, f(1^-) $\to -\infty$
As $x \to 1^+$, f(1^-) $\to \infty$
So, the limit *does not exist*.

Using the same f(x) there is an issue with f(x), when x = -1, this is because f(-1) = $\frac{(-1+1)^2}{(-1-1)(-1+1)} = \frac{0}{0} = ?$

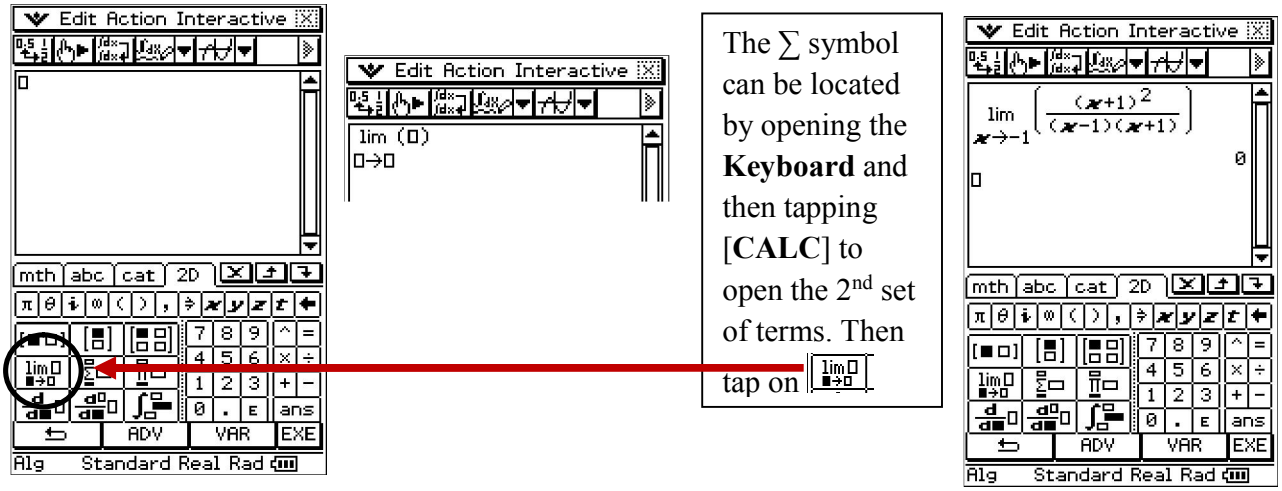
So how can we determine if there is a limit as $x \to -1$?

We could look to see what happens by selecting values of x close to x = -1.



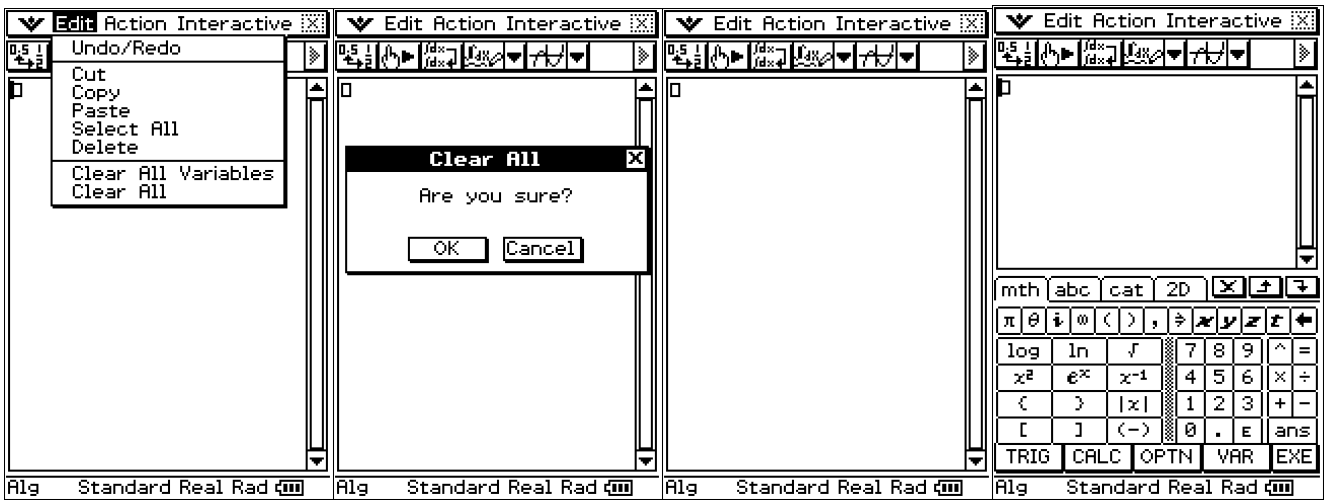
This shows that as the values of $x \to -1$. There is no limit.
As $x \to -1^-$, f(-1^-) $\to 0$
As $x \to -1^+$, f(-1^+) $\to 0$
So, the limit *does exis and equals 0*.

Exploring the use of the **2D** notation on the Classpad: '∑'



The ∑ symbol can be located by opening the **Keyboard** and then tapping [**CALC**] to open the 2nd set of terms. Then tap on $\lim_{\square\to\square}$

Tip: It is always better to start with a 'clean' working window on the ClassPad when exploring new concepts.



Tap [**Edit**] | Tap [**OK**] | A 'clean' working window | Press [**Keyboard**]