

# Area Animations

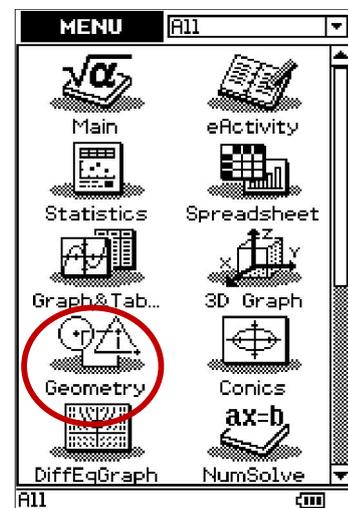
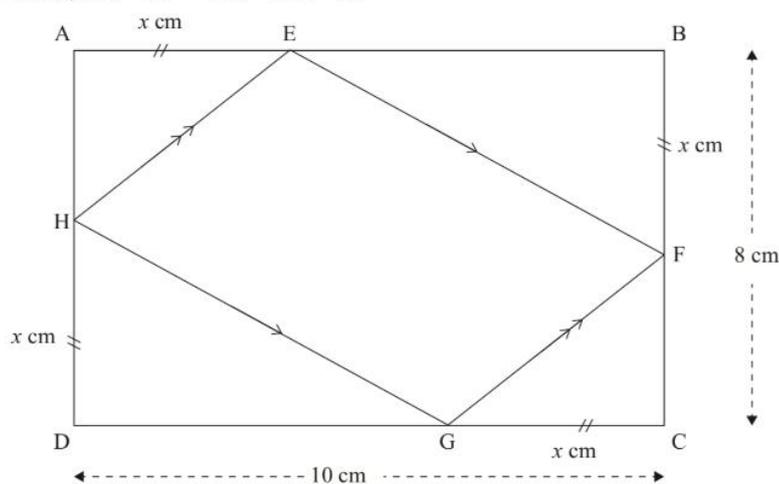
*This resource was written by Derek Smith with the support of CASIO New Zealand. It may be freely distributed but remains the intellectual property of the author and CASIO.*

**This activity was inspired by the 2018 NCEA Level 2 calculus paper.**

A rectangle ABCD measures 10 cm by 8 cm. A parallelogram EFGH can be drawn inside the rectangle, as shown in the diagram below.

Suppose that the distance from each corner of the rectangle to the next vertex of the parallelogram, in a clockwise direction, is  $x$  cm.

That is,  $AE = BF = CG = DH = x$ .



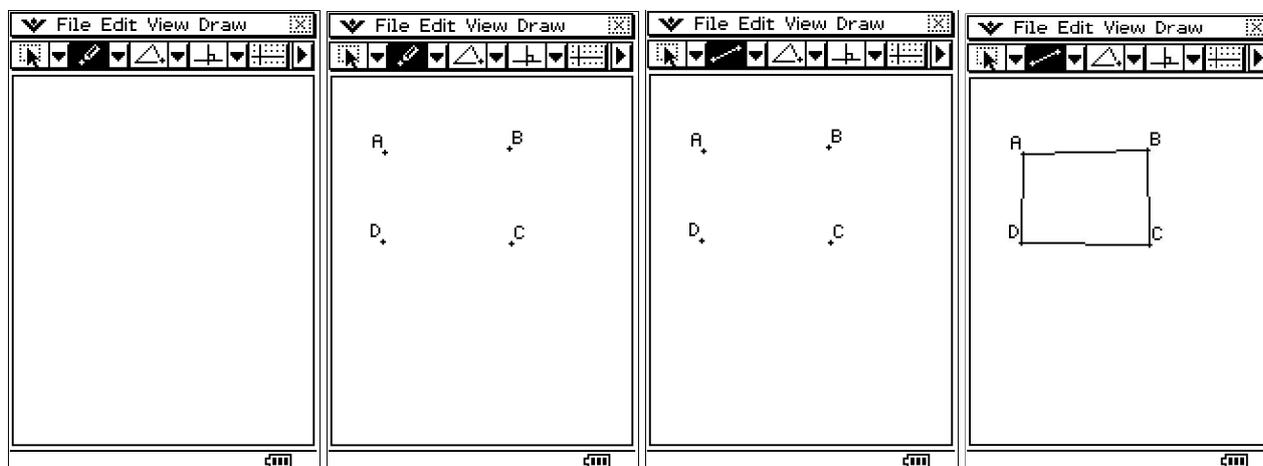
Select the **Geometry** Icon from the **MAIN MENU**.

### Construction:

To animate points on a line (or curve) the direction is important so that the point starts and finished in the appropriate positions. For example constructing a line that starts at point A and ends at point B will mean that a point on the line AB can be animated and by default the animated point starts at point A and moves towards point B, ending the animation at point B.

To create the Rectangle ABCD:

1. Select the point icon, then construct the points A, B, C and D
2. Select the line segment icon, then connect A to B, B to C, C to D and D to A.



The lengths of the sides of the rectangle need to be set to fixed lengths and the vertices to be set to right angles.

Place the pointer in 'rest' mode on the top left, , then select page 2 of the geometry icons by tapping on the

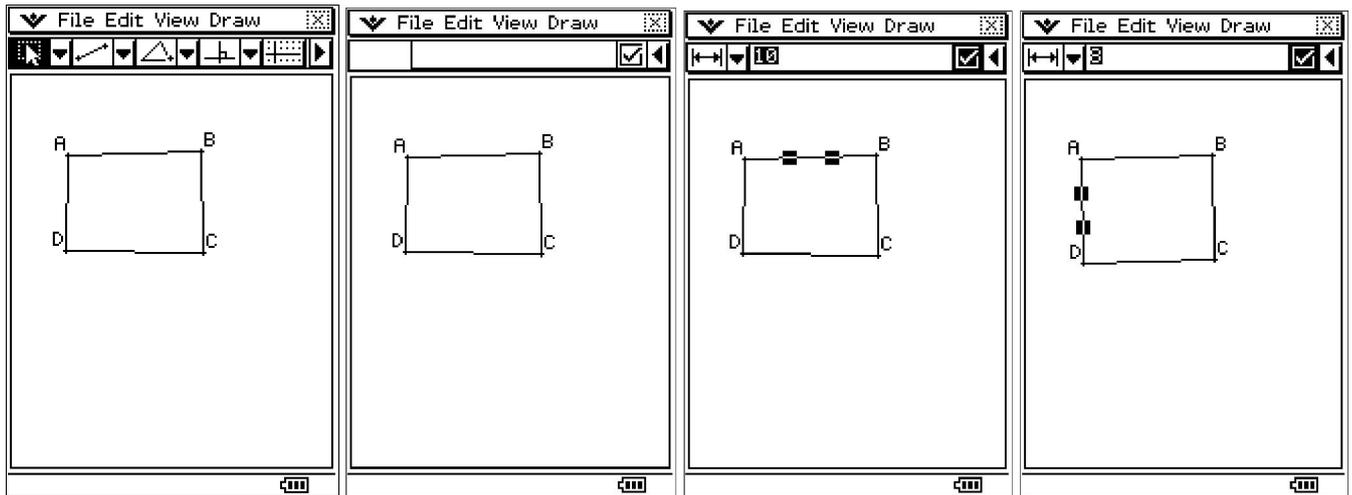
arrow, on the top right, .

Then tap on the line AB and set its length to '10'.

Tap on the line BC and set its length to '8'.

Tap on the line CD and set its length to '10'.

Tap on the line AD and set its length to '8'.



Now, tap on the line AB and BC, this will highlight the two lines and will indicate the angle between the lines, set this to 90.

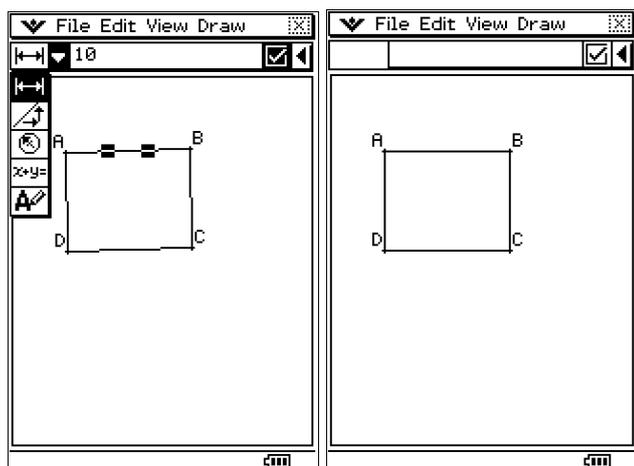
Tap on the line BC and CD, and set this to 90.

Tap on the line CD and DA, and set this to 90.

Tap on the line DA and AB, and set this to 90.

Select the dropdown for gradient, .

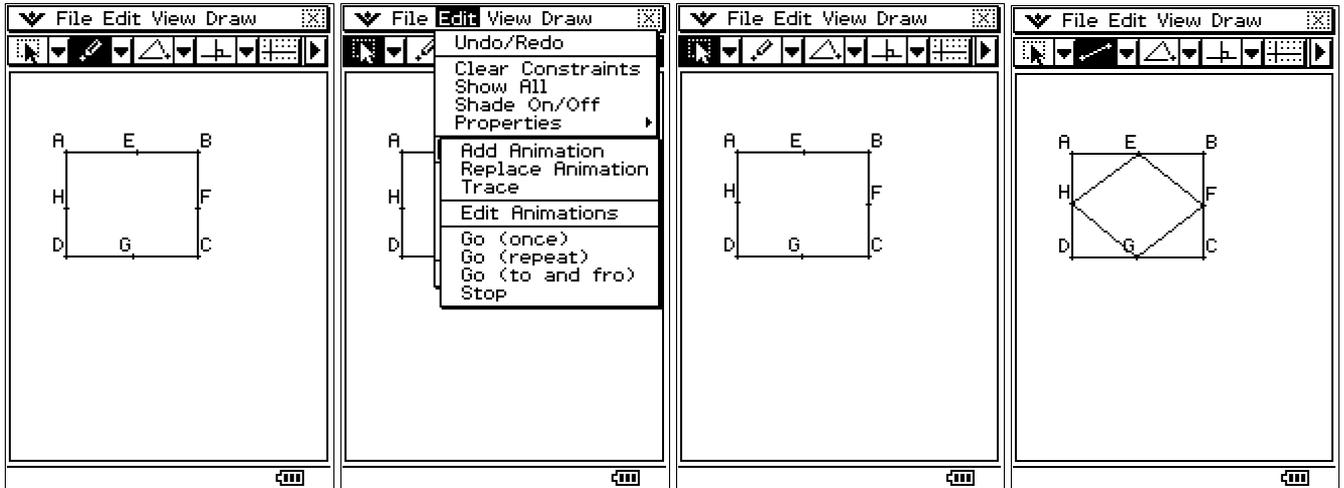
Select the line segment AB, and enter 0. [This will make the line segment AB to be horizontal and hence the other lines to form a rectangle]



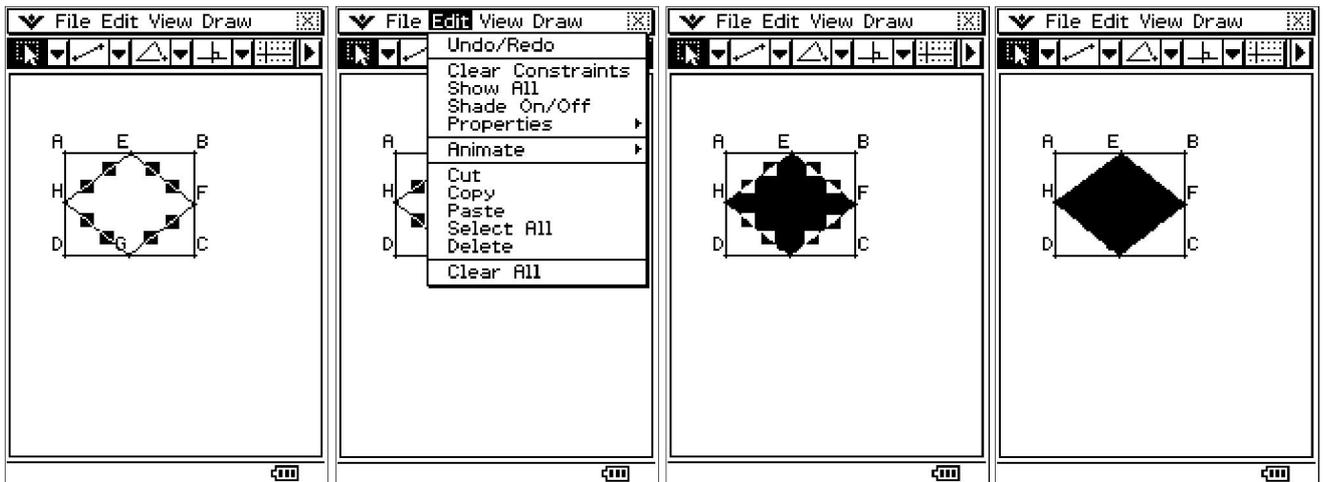
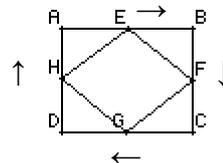
You have now created the rectangle ABCD. ☺

Now, select the point icon and place the points E, F, G and H on the appropriate line segments.

Now, animate each of the points E, F, G and H, one at a time by selecting the point and line to be animated on and then select 'Edit' and Animate, then 'Add Animation'. When the animations are complete, select the line segment icon and create the parallelogram EFGH.

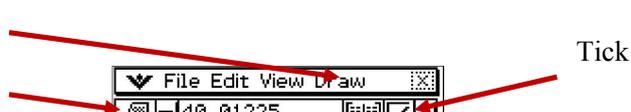


Check to make sure the animation is doing what you want it to do, i.e.



### Capturing the data.

Grid  
Area

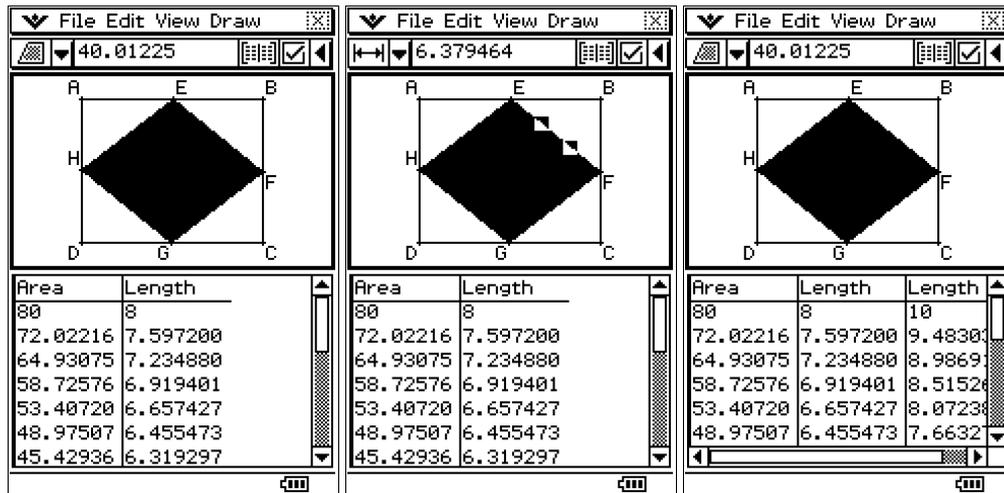


On the 2<sup>nd</sup> page of icons, note that there is a 'grid' icon, next to the 'tick'. Select the line segments EF, FG, GH and HE by tapping on them. This will highlight the AREA of the shaded shape. Now, tap on the 'grid' icon and you will see the area calculations for the parallelogram EFGH in the animation captured in numerical form.

Now tap in the 'white space' to unselect and then tap on the line segment HE. Now, tap on the 'grid' icon and you will see the length calculations for HE in the animation captured in numerical form.

Again tap in the 'white space' to unselect and then tap on the line segment EF. Now, tap on the 'grid' icon and you will see the length calculations for EF in the animation captured in numerical form.

What do you notice?



You can scroll through the displayed numerical values of the animation to decide where the minimum area of the parallelogram is.

Note that this is not quite the same problem set in the **2018 NCEA Level 2 calculus paper**.

How can you construct an animation to make it the same?

**Hint:** All four sides need to be the same length so that the point that is animated along each line segment travels the same distance i.e. from  $0 \rightarrow 8$  cm, not two covering  $0 \rightarrow 8$  cm and the other two covering  $0 \rightarrow 10$  cm

**The algebra:**

$$\begin{aligned}
 \text{Area of parallelogram (A)} &= \text{Area of rectangle} - \text{area of the four triangles formed between the rectangle and parallelogram} \\
 &= 10 \times 8 - 2 \times \frac{1}{2} (10 - x) \times x - 2 \times \frac{1}{2} (8 - x) \times x \\
 &= 80 - (10 - x) \times x - (8 - x) \times x \\
 &= 80 - 10x + x^2 - 8x + x^2 \\
 &= 80 - 18x + 2x^2
 \end{aligned}$$

Solving for minimum area (as is a positive quadratic) occurs when  $\frac{dA}{dx} = 0$ , i.e.  $-18 + 4x = 0$ ,  $x = 4\frac{1}{2}$

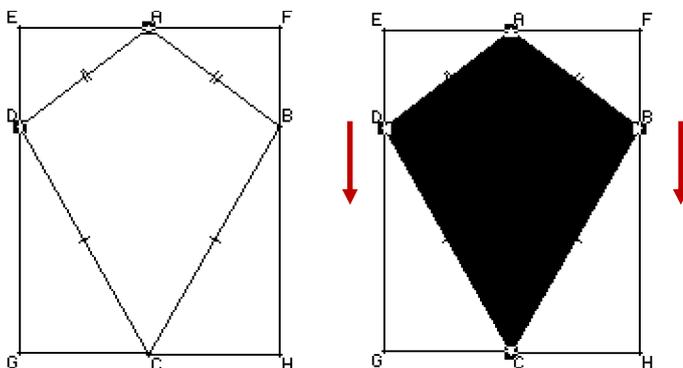
What is the algebra with a rectangle of size  $m \times n$ ?

$$\begin{aligned}
 \text{Area of parallelogram (A)} &= \text{Area of rectangle} - \text{area of the four triangles formed between the rectangle and parallelogram} \\
 &= m \times n - 2 \times \frac{1}{2} (m - x) \times x - 2 \times \frac{1}{2} (n - x) \times x \\
 &= m \times n - (m - x) \times x - (n - x) \times x \\
 &= m \times n - mx + x^2 - nx + x^2 \\
 &= m \times n - (m + n)x + 2x^2
 \end{aligned}$$

Solving for minimum area (as is a positive quadratic) occurs when  $\frac{dA}{dx} = 0$ , i.e.  $-(m + n) + 4x$ ,  $x = \frac{(m + n)}{4}$

**Other animations:**

Try creating this (a kite inside a rectangle) and gather data from the animations to confirm the area of the enclosed shapes.



Make up some other animations of your own.

For further tips, more information and software support visit our websites:

[www.casio.edu.monaco.orp.co.nz](http://www.casio.edu.monaco.orp.co.nz) or <http://graphic-technologies.co.nz>