Binomial – Poisson – Normal Distribution Approximations.

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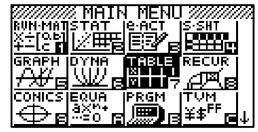
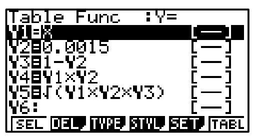
Select TABLE mode from the main menu by using the arrow keys to highlight the TABLE icon or by pressing 7.

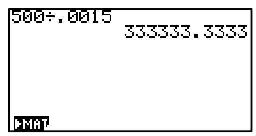


<u>Example:</u> An email message advertises the chance to win a prize if the reader follows a link to an on-line survey. The probability that a recipient of the email clicks on the link to the survey is 0.0015. How many emails, to the nearest hundred, need to be sent out in order to have a 99% probability that at least 500 will be answered?

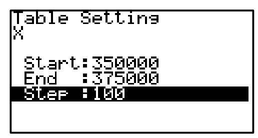
<u>Answer:</u> You want Prob(X=500,501,502,...,n) = 0.99
Perhaps an approximation of the Binomial by the Poisson Distribution would be helpful here, as p is small and N is large. As the numbers of emails that you are dealing with are very large and the probability of a response is small an approximation is necessary, as the use of ^nC_r will give you 'overflow' on the calculator. Hence a Poisson Distribution is appropriate in the first instance.
BUT... you will find that λ = n×p becomes very large for you to use either the tables or your calculator also! So, a Binomial Distribution approximation is now useful where:
μ = n×p and σ = √(n×p×(1-p))
You need to have μ > 500, so that you can get Prob(X=500,501,502,...,n) = 0.99

Have a:
Column for n
Column for p
Column for 1-p
Column for n×p
Column for √(n×p×(1-p))

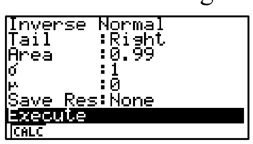


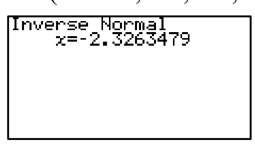


To find an approximate minimum value for n: 500/0.0015 = n_{min}
Hint: n > 333333, try using a Table to help with n incrementing by 100 in each cell.

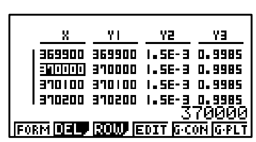


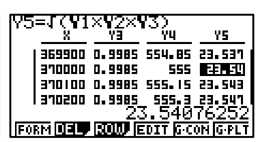
Using the Standard Normal Distribution model gives Prob(X=500,501,502,...,n) = 0.99 a z-score of -2.3263479.



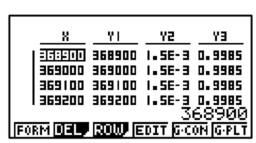


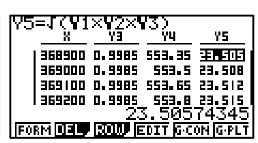
Hence we want the 'distance' from 499.5 [Continuity correction for 500.] to the mean to be σ×2.3263479
Scrolling down the list to 370000:
μ = n×p = 555 and σ = √(n×p×(1-p)) = 23.54076252
and using these values gives Prob(X=500,501,502,...,n) = Prob(X ≥ 500.5) = 0.99





Scrolling back up a little further...
μ = n×p = 553.35 and σ = √(n×p×(1-p)) = 23.50574345
and using these values gives Prob(X=500,501,502,...,n) = Prob(X ≥ 499.5) = 0.99





Hence, to the nearest 100, a total of approximately 368900 emails need to be sent, so that at least 500 will be answered with a probability of 0.99.