Normal – Binomial – Poisson Distribution Approximations.

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Select STAT mode from the main menu by using the arrow keys to highlight the STAT icon or by pressing 2.

Relevant Background Information

The Graphics calculator can give you the opportunity to calculate probability values from the Normal – Binomial – Poisson Distributions

Justifications for the:
1. Normal approximation to the Poisson
2. Normal approximation to the Binomial
3. Poisson approximation to the Binomial

requires a move from a discrete statistical distribution (Poisson and Binomial) to a continuous statistical distribution (Normal), and hence a 'continuity correction' is required.
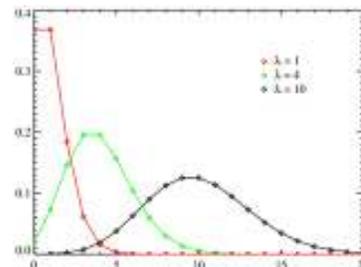
Source: http://en.wikipedia.org/wiki/Poisson_distribution

'Rule of Thumb'

Normal approximation to the Poisson, here if $\lambda > 20$, then $\mu = \lambda$ and $\sigma = \sqrt{\lambda}$.

Normal approximation to the Binomial, here the population value, n, needs to be large, along with $n \times p > 5$ and $n \times (1 – p) > 5$.

Poisson approximation to the Binomial, here the population value, n, needs to be large, along p being small.

discrete random variable

23 24 25

continuous random variable
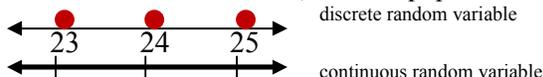
Example: For a discrete random variable Prob$(23 < X \le 25)$ = Prob(X = 24 and 25) to convert to a continuous random variable Prob$(23 < X \le 25)$ becomes Prob$(23.5 \le X \le 25.5)$
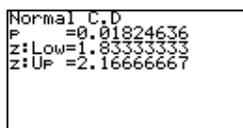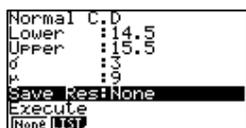
Example 1:   Normal approximation to the Poisson

Historically, we expect 9 fatal accidents in any given week in NZ on average. Last week, there were 15 fatal accidents. How likely is this? Using the Poisson Distribution:

Number of accidents	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Probability %	0	0.1	0.5	1.5	3.4	6.1	9.1	11.7	13.2	13.2	11.9	9.7	7.3	5	3.2	1.9	1.1	0.6	0.3	0.1

The probability of 15 fatalities is approximately 1.9%. The probability of getting 15 or more events is 1.9% + 1.1% + 0.6% + 0.3% + 0.1% or about 4.0%.

Use the Normal approximation to the Poisson: A Poisson variable with a mean of N is distributed approximately like a Normal variable with a mean of N and a SD of $\sqrt{N}$. A Poisson variable with a mean of 9 is distributed like a Normal variable with a mean of 9 and a SD of 3.

Answer: Prob(15 fatalities) = Prob$(14.5 \le X \le 15.5)$
= 1.825%
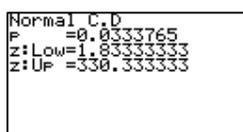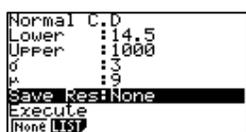
Answer: Prob(15 or more fatalities) = Prob$(14.5 \le X)$
= 3.34%

Example 2:   Normal approximation to the Binomial

A fair coin is tossed 36 times. Find the probability of gaining between 15 to 25 successful heads inclusive, i.e. Prob$(15 \le X \le 25)$. Let X be the total number of tails. Then X is a binomial random variable with mean 18 $(n \times p = 36 \times ½)$ and standard deviation 3 $(\sqrt{(n \times p \times (1-p)} = \sqrt{(36 \times ½ \times ½)})$.

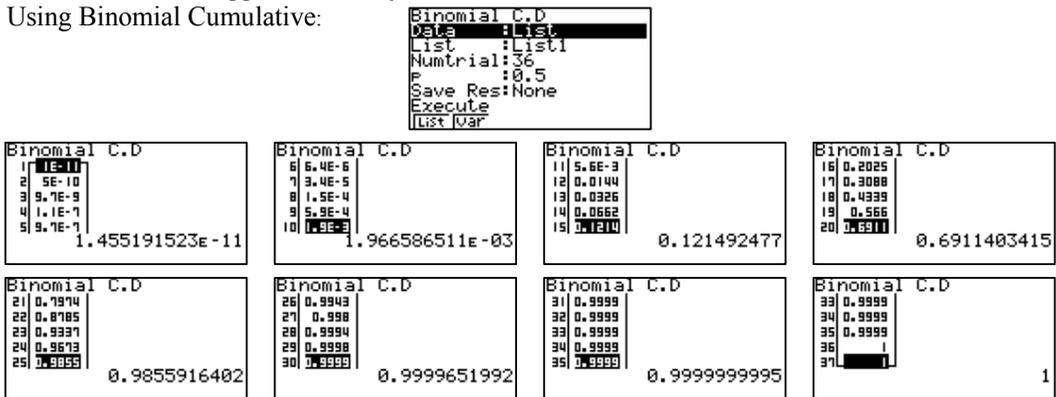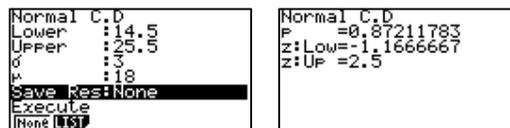|			
Binomial P.D 1 1E-11 2 5E-10 3 9.1E-9 4 1E-7 5 8.5E-7 5.238689482ᴇ-10	Binomial P.D 6 5.4E-6 7 2.8E-5 8 1.2E-4 9 4.4E-4 10 1.3E-3 1.36996503ᴇ-03	Binomial P.D 11 3.6E-3 12 8.7E-3 13 0.0182 14 0.0336 15 0.0552 0.05524339504	Binomial P.D 16 0.081 17 0.1063 18 0.1251 19 0.132 20 0.1251 0.1251100417
Prob(X=1) = 0.0000	Prob(X=9) = 0.0014	Prob(X=14) = 0.0552	Prob(X=19) = 0.1251
Binomial P.D 21 0.1063 22 0.081 23 0.0552 24 0.0336 25 0.0182 0.01821430778	Binomial P.D 26 8.7E-3 27 3.6E-3 28 1.3E-3 29 4.4E-4 30 1.2E-4 1.214747317ᴇ-04	Binomial P.D 31 2.8E-5 32 5.4E-6 33 8.5E-7 34 1E-7 35 9.1E-9 9.167706594ᴇ-09	Binomial P.D 33 8.5E-7 34 1E-7 35 9.1E-9 36 5E-10 37 1E-11 1.455191523ᴇ-11
Prob(X=24) = 0.0182	Prob(X=29) = 0.0001	Prob(X=34) = 0.0000	Prob(X=36) = 0.0000

We have from the individual table values illustrated above: Prob(X=15) = 0.081; Prob(X=16) = 0.1063; Prob(X=17) = 0.1251; Prob(X=18) = 0.132; P(X=19)=0.1251; Prob(X=20) = 0.1063; Prob(X=21) = 0.1081; Prob(X=22) = 0.0522; Prob(X=23) = 0.0336; Prob(X=24) = 0.0182; Prob(X=25) = 0.0087; The binomial distribution can be approximated by the normal distribution with mean of 18 and deviation of 3.

Using Binomial Cumulative:

```
Binomial C.D
Data    :List
List    :List1
Numtrial:36
P       :0.5
Save Res:None
Execute
List Var
```

Binomial C.D 1 1E-11 2 5E-10 3 9.1E-9 4 1.1E-7 5 9.1E-7 1.455191523ᴇ-11	Binomial C.D 6 6.4E-6 7 3.4E-5 8 1.5E-4 9 5.9E-4 10 1.9E-3 1.966586511ᴇ-03	Binomial C.D 11 5.6E-3 12 0.0144 13 0.0326 14 0.0662 15 0.1214 0.121492477	Binomial C.D 16 0.2025 17 0.3088 18 0.4339 19 0.566 20 0.6911 0.6911403415
Binomial C.D 21 0.7974 22 0.8785 23 0.9337 24 0.9673 25 0.9855 0.9855916402	Binomial C.D 26 0.9943 27 0.998 28 0.9994 29 0.9998 30 0.9999 0.9999651992	Binomial C.D 31 0.9999 32 0.9999 33 0.9999 34 0.9999 35 0.9999 0.9999999995	Binomial C.D 33 0.9999 34 0.9999 35 0.9999 36 1 37 1 1

Prob($15 \leq X \leq 25$) = 0.9943345079 - 0.121492477 = 0.8728420309.
The area under the normal curve is 87%.

Using the Normal distribution:

```
Normal C.D
Lower   :14.5
Upper   :25.5
σ       :3
μ       :18
Save Res:None
Execute
None LIST
```
```
Normal C.D
P    =0.87211783
z:Low=-1.1666667
z:Up =2.5
```

Example 3:   Poisson approximation to the Binomial

During the lunch hour, 47 customers will walk through the doors of the post office. Assume that each customer arrives at a random time and independent of any other customer. What is the probability that more than one person walks through the door during the first minute of lunch hour?

The probability that they choose the first minute is then $^1/_{60}$. Thus, we have n = 47 repetitions of a trial, the expected number of success is: λ = n×p = 47×$^1/_{60}$ = 0.783 (3d.p.).
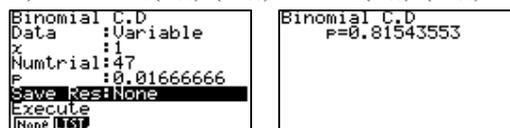
Prob(more than 1 arrival) = 1 – P(0 or 1 arrival).
Prob(0 or 1 arrival in 1st minute) = $0.783^0 e^{0.783}1.783/0! + 0.783^1 e^{0.783}/1!$ = 0.815
Prob (more than 1 arrival in 1st minute) = 1 - 0.815 = 0.185.

Using the Binomial distribution:
Prob(more than 1 arrival) = 1 – $^{47}C_0(^1/_{60})^0(^{59}/_{60})^{47} + {}^{47}C_1(^1/_{60})^1(^{59}/_{60})^{46}$ = 1 – 0.815 = 0.185

```
Binomial C.D
Data    :Variable
x       :1
Numtrial:47
P       :0.01666666
Save Res:None
Execute
None LIST
```
```
Binomial C.D
P=0.81543553
```

For some interactive Normal, Binomial and Poisson Distribution applets to view:
http://www.waldomaths.com/Poisson1NL.jsp and
http://www.waldomaths.com/Binomial1NL.jsp and
http://www.waldomaths.com/PoissBin1NL.jsp